1. Harap palajari diskusikan dan kerjakan soal-soal yang ada pada akhir materi yang terlampir pada FILE.

2. Selanjutnya menjawab sebelum batas waktu yang sudah ditentukan.

DERET TAYLOR DAN MAC LAURIN

1. a. $f(x) = \frac{1}{1+x}$

$f(x) = \frac{1}{1+x} \rightarrow f(0) = \frac{1}{1+0} = 1 \rightarrow f(c) = \frac{1}{1+c} = (1+c)^{-1}$

$f'(x) = -\frac{1}{(1+x)^2} \rightarrow f'(0) = -\frac{1}{(1+0)^2} = -1 \rightarrow f'(c) = -\frac{1}{(1+c)^2} = -(1+c)^{-2}$

$f''(x) = \frac{2}{(1+x)^3} \rightarrow f''(0) = \frac{2}{(1+0)^3} = 2 \rightarrow f''(c) = \frac{2}{(1+c)^3} = 2.(1+c)^{-3}$

$f'''(x) = -\frac{6}{(1+x)^4} \rightarrow f'''(0) = -\frac{6}{(1+0)^4} = -6 \rightarrow f'''(c) = -\frac{6}{(1+c)^4} = -6.(1+c)^{-4}$

$sehingga\ deret\ Mc\ Laurin\ \rightarrow \frac{1}{1+x} = 1 - x + \frac{2}{2!}x^2 - \frac{6}{3!}x^3 + \cdots + \frac{(-1)^n.\cancel{n!}}{\cancel{n!}}x^n$

$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \cdots + (-x)^n$

$deret\ Taylor:$

$\frac{1}{1+x} = (1+c)^{-1} - \frac{(1+c)^{-2}}{1!}(x-c) + \frac{2.(1+c)^{-3}}{2!}(x-c)^2 - \frac{6.(1+c)^{-4}}{3!}(x-c)^3 + \cdots + \frac{(-1)^n.\cancel{n!}.(1+c)^{-(n+1)}}{\cancel{n!}}(x-c)^n$

$\frac{1}{1+x} = (1+c)^{-1} - \frac{(x-c)}{(1+c)^2} + \frac{(x-c)^2}{(1+c)^3} - \frac{(x-c)^3}{(1+c)^4} + \cdots + \frac{(-1)^n(x-c)^n}{(1+c)^{(n+1)}}$

b. $f(x) = \ln(1+x)$

$f(x) = \ln(1+x) \rightarrow f(0) = \ln(1+0) = 0 \rightarrow f(c) = \ln(1+c)$

$f'(x) = \frac{1}{1+x} \rightarrow f'(0) = \frac{1}{1+0} = 1 \rightarrow f'(c) = \frac{1}{1+c} = (1+c)^{-1}$

$f''(x) = -\frac{1}{(1+x)^2} \rightarrow f''(0) = -\frac{1}{(1+0)^2} = -1 \rightarrow f''(c) = -\frac{1}{(1+c)^2} = -(1+c)^{-2}$

$f'''(x) = \frac{2}{(1+x)^3} \rightarrow f'''(0) = \frac{2}{(1+0)^3} = 2 \rightarrow f'''(c) = \frac{2}{(1+c)^3} = 2.(1+c)^{-3}$

$f^4(x) = -\frac{6}{(1+x)^4} \rightarrow f^4(0) = -\frac{6}{(1+0)^4} = -6 \rightarrow f^4(c) = -\frac{6}{(1+c)^4} = -6.(1+c)^{-4}$

$sehingga\ deret\ Mc\ Laurin\ \rightarrow \ln(1+x) = 0 + \frac{1}{1!}x - \frac{1}{2!}x^2 + \frac{2}{3!}x^3 - \frac{6}{4!}x^4 + \cdots$

$\ln(1+x) = 0 + x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + \frac{(-1)^{n+1}}{n}x^n$

$deret\ Taylor:$

$\ln(1+x) = \ln(1+c) + \frac{(x-c)}{1!(1+c)} - \frac{(x-c)^2}{2!(1+c)^2} + \frac{2.(x-c)^3}{3!(1+c)^3} - \frac{6.(x-c)^4}{4!(1+c)^4} + \cdots$

$\ln(1+x) = \ln(1+c) + \frac{(x-c)}{(1+c)} - \frac{(x-c)^2}{2.(1+c)^2} + \frac{(x-c)^3}{3.(1+c)^3} - \frac{(x-c)^4}{4.(1+c)^4} + \cdots + \frac{(-1)^{n+1}.(x-c)^n}{n.(1+c)^n}$

2. a. $f(x) = \sin x$

$f(x) = \sin x \rightarrow f(0) = \sin(0) = 0 \rightarrow f(c) = \sin(c)$

$f'(x) = \cos x \rightarrow f'(0) = \cos(0) = 1 \rightarrow f'(c) = \cos(c)$

$f''(x) = -\sin x \rightarrow f''(0) = -\sin(0) = 0 \rightarrow f''(c) = -\sin(c)$

$f'''(x) = -\cos x \rightarrow f'''(0) = -\cos(0) = -1 \rightarrow f'''(c) = -\cos(c)$

$f^4(x) = \sin x \rightarrow f^4(0) = \sin(0) = 0 \rightarrow f^4(c) = \sin(c)$

$f^5(x) = \cos x \rightarrow f^5(0) = \cos(0) = 1 \rightarrow f^5(c) = \cos(c)$

$sehingga\ deret\ Mc\ Laurin\ \rightarrow \sin x = 0 + \frac{1}{1!}x - \frac{0}{2!}x^2 - \frac{1}{3!}x^3 + \frac{0}{4!}x^4 + \frac{1}{5!}x^5 + \cdots$

$\sin x = \frac{1}{1!}x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \cdots + \frac{(-1)^n.x^{2n+1}}{(2n+1)!}$

$deret\ Taylor:$

$\sin x = \sin(c) + \frac{\cos(c).(x-c)}{1!} - \frac{\sin(c).(x-c)^2}{2!} - \frac{\cos(c).(x-c)^3}{3!} + \cdots + \frac{(-1)^n.\sin(c).(x-c)^{2n}}{(2n)!} + \frac{(-1)^n.\cos(c).(x-c)^{2n+1}}{(2n+1)!}$

$b. f(x) = \cos x$

$f(x) = \cos x \ \rightarrow f(0) = \cos(0) = 1 \rightarrow f(c) = \cos(c)$

$f'(x) = -\sin x \ \rightarrow f'(0) = -\sin(0) = 0 \rightarrow f'(c) = -\sin(c)$

$f''(x) = -\cos x \ \rightarrow f''^{(0)} = -\cos(0) = -1 \rightarrow f''(c) = -\cos(c)$

$f'''(x) = \sin x \ \rightarrow f'''(0) = \sin(0) = 0 \rightarrow f'''(c) = \sin(c)$

$f^4(x) = \cos x \ \rightarrow f^4(0) = \cos(0) = 1 \rightarrow f^4(c) = \cos(c)$

$f^5(x) = -\sin x \ \rightarrow f^5(0) = -\sin(0) = 0 \rightarrow f^5(c) = -\sin(c)$

$f^6(x) = -\cos x \ \rightarrow f^6(0) = -\cos(0) = -1 \rightarrow f^6(c) = -\cos(c)$

$sehingga\ deret\ Mc\ Laurin\ \rightarrow \cos x = 1 + \frac{0}{1!}x - \frac{1}{2!}x^2 + \frac{0}{3!}x^3 + \frac{1}{4!}x^4 + \frac{0}{5!}x^5 - \frac{1}{6!}x^6 + \cdots$

$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \cdots + \frac{(-1)^n . x^{2n}}{(2n)!}$

$deret\ Taylor$:

$\cos x = \cos(c) - \frac{\sin(c) . (x-c)}{1!} - \frac{cos(c) . (x-c)^2}{2!} + \frac{\sin(c) . (x-c)^3}{3!} + \cdots + \frac{(-1)^n . \cos(c) . (x-c)^{2n}}{(2n)!} - \frac{(-1)^n . \sin(c) . (x-c)^{2n+1}}{(2n+1)!}$

PART 7 PERSAMAAN DIFFERENSIAL HOMOGEN ORDO KE-DUA

MATEMATIKA TERAPAN (MTS 271101)

Oleh

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PROGRAM PASCASARJANA MAGISTER TEKNIK SIPIL

UNIVERSITAS BINA DARMA

2020

2. $y" + 5y' - 6y = 0$

$r^2 + 5r - 6 = 0, a = 1, b = 5, c = -6$
$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \to r = \frac{-5 \pm \sqrt{5^2 - 4.1.(-6)}}{2.1} \to r = \frac{-5 \pm \sqrt{25 + 24}}{2} \to r = \frac{-5 \pm 7}{2}$$
$$r_1 = \frac{-5 + 7}{2} = 1, r_2 = \frac{-5 - 7}{2} = -6$$
$y = C_1.e^x + C_2.e^{-6x}$

3. $y'' + 6y' - 7y = 0, y = 0, y' = 4\ pada\ x = 0 \to y(0) = 0, y'(0) = 4$

$r^2 + 6r - 7 = 0, a = 1, b = 6, c = -7$
$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \to r = \frac{-6 \pm \sqrt{6^2 - 4.1.(-7)}}{2.1} \to r = \frac{-6 \pm \sqrt{36 + 28}}{2} \to r = \frac{-6 \pm 8}{2}$$
$$r_1 = \frac{-6 + 8}{2} = 1, r_2 = \frac{-6 - 8}{2} = -7$$
$y = C_1.e^x + C_2.e^{-7x} \to y(0) = 0 \to 0 = C_1.e^0 + C_2.e^{-7.0} \to C_1 + C_2 = 0 \to C_2 = -C_1$
$y' = C_1.e^x - 7.C_2.e^{-7x} \to y'(0) = 4 \to 4 = C_1.e^0 - 7.(-C_1).e^{-7.0} \to C_1 + 7.C_1 = 4$
$$\to C_1 = \frac{1}{2}$$
$$y = \frac{1}{2}.e^x - \frac{1}{2}.e^{-7x}$$

5. $y'' - 4y' + 4y = 0$

$r^2 - 4r + 4 = 0, a = 1, b = -4, c = 4$
$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \to r = \frac{-(-4) \pm \sqrt{(-4)^2 - 4.1.4}}{2.1} \to r = \frac{4 \pm \sqrt{16 - 16}}{2} \to r = \frac{4 \pm 0}{2}$$
$$r_1 = \frac{4 + 0}{2} = 2, r_2 = \frac{4 - 0}{2} = 2 \to akar\ berulang$$
$y = C_1.e^{2x} + C_2.e^{2x}$

8. $y'' + 6y' - 2y = 0$

$r^2 + 6r - 2 = 0, a = 1, b = 6, c = -2$
$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \to r = \frac{-6 \pm \sqrt{(6)^2 - 4.1.(-2)}}{2.1} \to r = \frac{-6 \pm \sqrt{36 + 8}}{2} \to r = \frac{-6 \pm 2\sqrt{7}}{2}$$
$$r_1 = \frac{-6 + 2\sqrt{7}}{2} = -3 + \sqrt{7}, r_2 = \frac{-6 - 2\sqrt{7}}{2} = -3 - \sqrt{7}$$
$y = C_1.e^{(-3+\sqrt{7})x} + C_2.e^{(-3-\sqrt{7})x}$

10. $y'' + 9y = 0, y = 3, y' = 3\ pada\ x = \frac{\pi}{3} \rightarrow y\left(\frac{\pi}{3}\right) = 3, y'(\frac{\pi}{3}) = 3$

$$r^2 + 9 = 0, a = 1, b = 0, c = 9$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow r = \frac{-0 \pm \sqrt{(0)^2 - 4.1.9}}{2.1} \rightarrow r = \frac{-0 \pm \sqrt{0 - 36}}{2} \rightarrow r = \frac{0 \pm 6i}{2}$$

$$= 0 \pm 3i$$

$$\alpha = 0, \beta = 3 \rightarrow y = C_1.e^{\alpha x}.\cos\beta x + C_2.e^{\alpha x}.\sin\beta x \rightarrow y = C_1.e^0.\cos 3x + C_2.e^0.\sin 3x$$

$$y = C_1.\cos 3x + C_2.\sin 3x \rightarrow \mathrm{y}\left(\frac{\pi}{3}\right) = 3 \rightarrow 3$$

$$= C_1.\cos\left(3.\frac{\pi}{3}\right) + C_2.\sin\left(3.\frac{\pi}{3}\right) \rightarrow -C_1 + 0 = 3 \rightarrow C_1 = -3$$

$$y' = C_1.-\frac{1}{3}.\sin 3x + C_2.\frac{1}{3}.\cos 3x \rightarrow y' = -\frac{C_1}{3}.\sin 3x + \frac{C_2}{3}.\cos 3x$$

$$\mathrm{y}'^{\left(\frac{\pi}{3}\right)} = 3 \rightarrow 3 = -\frac{-3}{3}.\sin\left(3.\frac{\pi}{3}\right) + \frac{C_2}{3}.\cos\left(3.\frac{\pi}{3}\right) \rightarrow \frac{C_2}{3}.(-1) = 3 \rightarrow C_2 = -9$$

$$y = -3.\cos 3x - 9.\sin 3x$$

12. $y'' + y' + y = 0$

$$r^2 + r + 1 = 0, a = b = c = 1$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow r = \frac{-1 \pm \sqrt{(1)^2 - 4.1.1}}{2.1} \rightarrow r = \frac{-1 \pm \sqrt{1 - 4}}{2} \rightarrow r = \frac{-1 \pm \sqrt{3}i}{2}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$$

$$\alpha = -\frac{1}{2}, \beta = \frac{\sqrt{3}}{2} \rightarrow y = C_1.e^{\alpha x}.\cos\beta x + C_2.e^{\alpha x}.\sin\beta x$$

$$y = C_1.e^{-\frac{1}{2}x}.\cos\frac{\sqrt{3}}{2}x + C_2.e^{-\frac{1}{2}x}.\sin\frac{\sqrt{3}}{2}x$$