1. Silakan palajari , diskusikan dan kerjakan soal-soal yang ada pada akhir materi yang terlampir.

2. Kemudian jawaban dikirimkan sebelum batas waktu yang sudah ditentukan.

NAMA : HENDI

① $y' = 2xy$

$2x = \frac{y'}{xy}$

$y'' = 2Ay + 2Axy'$

$= 2A(y + xy')$

$= \frac{y'}{xy}(y + xy')$

Jadi : $xyy'' - yy' - x(y')^2 = 0.$

② $\frac{dy}{dx} = e^{-y}(2x - y)$

$x - y = v \rightarrow 1 - \frac{dy}{dx} = \frac{dv}{dx} = \frac{dy}{dx} = e^{-y}\frac{dv}{dx}.$

$\rightarrow e^{-y} - \frac{dv}{dx} = 2 - \frac{dy}{dx} = \frac{dv}{dx} = 2 - \frac{dv}{dx}$

$e^{-v} = x + c$

$= e^{y-v} - x + c.$

⑤ $\sec x \cos^2 y \, dx = \cos x \sin y \, dy.$

PART 6 PERSAMAAN DIFFERENSIAL

MATEMATIKA TERAPAN (MTS 271101)

Oleh

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Dosen Program	**: Dr. H. Jemakmun, M.Si.**

PROGRAM PASCASARJANA MAGISTER TEKNIK SIPIL

UNIVERSITAS BINA DARMA

2020

1. $y' = 2xy$

$$\frac{dy}{dx} = 2xy \rightarrow dy = 2xy\,dx \rightarrow \frac{dy}{y} = 2x\,dx \rightarrow y^{-1}dy - 2x\,dx = 0$$

$$\int y^{-1}dy - \int 2x\,dx = \int 0 \rightarrow \ln|y| - x^2 = c$$

2. $\frac{dy}{dx} = e^{-y}.(2x-4)$

$$\frac{dy}{e^{-y}} = (2x-4)dx \rightarrow e^{y}\,dy = (2x-4)dx \rightarrow e^{y}\,dy - (2x-4)dx = 0$$

$$\int e^{y}\,dy - \int (2x-4)dx = \int 0 \rightarrow e^{y} - x^2 + 4x = c$$

3. $\frac{6.dr}{d\theta} = \frac{r^2}{\theta}$

$$\frac{6.dr}{r^2} = \frac{d\theta}{\theta} \rightarrow 6.r^{-2}dr = \theta^{-1}\,d\theta \rightarrow \theta^{-1}\,d\theta - 6.r^{-2}dr = 0$$

$$\int \theta^{-1}\,d\theta - \int 6.r^{-2}dr = \int 0 \rightarrow \ln|\theta| + \ln\left|e^{\frac{6}{r}}\right| = \ln|c| \rightarrow c = \theta.e^{\frac{6}{r}}$$

$$untuk\ r(1) = 2, c = 1.e^{\frac{6}{2}} \rightarrow c = 20$$

$$Sehingga\ \ PD\ menjadi:\ \theta.e^{\frac{6}{r}} - 20 = 0$$

4. $\frac{dy}{dt} = e^{y-t}.(\sec(y)).(1+t^2)$

$$\frac{dy}{dt} = \frac{e^{y}}{e^{t}}.\left(\frac{1}{\cos(y)}\right).(1+t^2) \rightarrow \frac{\cos(y)}{e^{y}}dy = \frac{(1+t^2)}{e^{t}}dt$$

$$\rightarrow e^{-y}.\cos(y)\,dy - e^{-t}.(1+t^2)dt = 0$$

$$\int e^{-y}.\cos(y)\,dy - \int e^{-t}.(1+t^2)dt = \int 0$$

$$\int e^{-y}.\cos(y)\,dy \rightarrow u = e^{-y}, du = -e^{-y}dy, dv = \cos(y)\,dy, v = \int \cos(y)\,dy = \sin(y)$$

$$\int e^{-y}.\cos(y)\,dy = e^{-y}.\sin(y) - \int \sin(y).-e^{-y}dy = e^{-y}.\sin(y) + \int \sin(y).e^{-y}dy$$

$$\int \sin(y).e^{-y}dy \rightarrow u = e^{-y}, du = -e^{-y}dy, dv = \sin(y)\,dy, v = \int \sin(y)\,dy = -\cos(y)$$

$$\int \sin(y).e^{-y}dy = e^{-y}.-\cos(y) - \int -\cos(y).-e^{-y}d\,y$$

$$= -e^{-y}.\cos(y) - \int e^{-y}.\cos(y)\,dy$$

$$\int e^{-y}.\cos(y)\,dy = e^{-y}.\sin(y) - e^{-y}.\cos(y) - \int e^{-y}.\cos(y)\,dy$$

$$2.\int e^{-y}.\cos(y)\,dy = e^{-y}(\sin(y) - \cos(y)) \rightarrow \int e^{-y}.\cos(y)\,dy$$

$$= \frac{e^{-y}}{2}(\sin(y) - \cos(y))$$

$$\int e^{-t}.(1+t^2)dt \rightarrow u = 1+t^2, du = 2t\,dt, dv = e^{-t}dt, v = \int e^{-t}dt = -e^{-t}$$

$$\int e^{-t}.(1+t^2)dt = (1+t^2).-e^{-t} - \int -e^{-t}.2t\,dt = -e^{-t}.(1+t^2) + 2.\int e^{-t}.t\,dt$$

$$\int e^{-t}.t\,dt \rightarrow u = t, du = dt, dv = e^{-t}dt, v = \int e^{-t}dt = -e^{-t}$$

$$\int e^{-t}.t\,dt = t.-e^{-t} - \int -e^{-t}dt = -t.e^{-t} - e^{-t} = -e^{-t}(t+1)$$

$$\int e^{-t}.(1+t^2)dt = -e^{-t}.(1+t^2) - 2.e^{-t}(1+t) = -e^{-t}(1+t^2+2+2t)$$

$$= -e^{-t}(t^2+2t+3)$$

$$\int e^{-y}.\cos(y)\,dy - \int e^{-t}.(1+t^2)dt = c \rightarrow \frac{e^{-y}}{2}(\sin(y)-\cos(y)) + e^{-t}(t^2+2t+3)$$

$$= c$$

5. $8.\sec x\,.\cos^2 y\,dx = \cos x \sin y\,dy$

$$8.\frac{1}{\cos x}.\cos^2 y\,dx = \cos x \sin y\,dy \rightarrow \frac{8}{\cos^2 x}dx = \frac{\sin y}{\cos^2 y}dy \rightarrow \frac{8}{\cos^2 x}dx - \frac{\sin y}{\cos^2 y}dy = 0$$

$$\int \frac{8}{\cos^2 x}dx - \int \frac{\sin y}{\cos^2 y}dy = \int 0 \rightarrow 8.\int \frac{1}{\cos^2 x}dx - \int \frac{\sin y}{\cos^2 y}dy = \int 0$$

$$\int \frac{1}{\cos^2 x}dx \rightarrow u = \tan x = \frac{\sin x}{\cos x} \rightarrow \frac{du}{dx} = \frac{u}{v} = \frac{u'v - v'u}{v^2}, u$$

$$= \sin x, u' = \cos x\,, v = \cos x, v' = -\sin x$$

$$\frac{du}{dx} = \frac{\cos x.\cos x - (-\sin x).\sin x}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \rightarrow du = \frac{1}{\cos^2 x}dx$$

$$\int \frac{1}{\cos^2 x}dx = \int du = u = \tan x$$

$$\int \frac{\sin y}{\cos^2 y}dy \rightarrow u = \cos y, \frac{du}{dy} = -\sin y \rightarrow du = -\sin y\,dy$$

$$\int \frac{\sin y}{\cos^2 y}dy = \int \frac{-1}{u^2}du = -\int u^{-2}du = -\frac{1}{-1}u^{-1} = \frac{1}{u} = \frac{1}{\cos y} = \sec y$$

$$8.\int \frac{1}{\cos^2 x}dx - \int \frac{\sin y}{\cos^2 y}dy = \int 0 \rightarrow 8.\tan x - \sec y = c$$

Nama: Andi Supriyadi
Angkatan: IV

2. $y'' + 5y' - 6y = 0$

$r^2 + 5r - 6 = 0$, $a = 1$, $b = 5$, $c = -6$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow r = \frac{-5 \pm \sqrt{5^2 - 4.1.(-6)}}{2.1} \rightarrow r = \frac{-5 \pm \sqrt{25+24}}{2}$ $r = \frac{-5 \pm 7}{2}$

$r_1 = \frac{-5+7}{2} = 1$, $r_2 = \frac{-5-7}{2} = -6$

$y = C_1.e^{x} + C_2.e^{-6x}$

3. $y'' + 6y' - 7y = 0$, $y = 0$, $y' = 4$ pada $x = 0 \rightarrow y(0) = 0$, $y'(0) = 4$

$r^2 + 6r - 7 = 0$, $a = 1$, $b = 6$, $c = -7$

$r = \frac{-b \pm \sqrt{b^2 + 4ac}}{2a}$ $r = \frac{-6 \pm \sqrt{6^2 - 4.1.(-7)}}{2.1} \rightarrow \frac{-6 \pm \sqrt{36+28}}{2} \rightarrow r = \frac{-6 \pm 8}{2}$

$r_1 = \frac{-6+8}{2} = 1$, $r_2 = \frac{-6-8}{2} = -7$

$y = C_1.e^{x} + C_2 e^{-7x} \rightarrow y(0) = 0 = C_1.e^{0} + C_2 e^{-7.0} \rightarrow C_1 + C_2 = 0 \rightarrow C_2 = -C_1$

$y' = C_1 e^{x} - 7.C_2.e^{-7x} \rightarrow y'(0) = 4 \rightarrow 4 = C_1.e^{0} - 7.(-C_1)e^{-7.0} \rightarrow C_1 + 7.C_1 = 4 \rightarrow C_1 = \frac{1}{2}$

$y = \frac{1}{2}.e^{x} - \frac{1}{2}.e^{-7x}$

5. $y'' - 4y' + 4y = 0$

$r^2 - 4r + 4 = 0$, $a = 1$, $b = -4$, $c = 4$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow r = \frac{-(-4) \pm \sqrt{(-4)^2 - 4.1.4}}{2.1} \rightarrow r = \frac{4 \pm \sqrt{16 - 16}}{2} = r = \frac{4 \pm 0}{2}$

$r_1 = \frac{4+0}{2} = 2$, $r_2 = \frac{4-0}{2} = 2 \rightarrow$ akar berulang

$y = C_1.e^{2x} + C_2 e^{2x}$

8. $y'' + 6y' - 2y = 0$

$r^2 + 6r - 2 = 0$, $a = 1$, $b = 6$, $c = -2$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow r = \frac{-6 \pm \sqrt{6^2 - 4.1.(-2)}}{2.1} \rightarrow r = \frac{-6 \pm \sqrt{36+8}}{2} \rightarrow r = \frac{-6 \pm 2\sqrt{7}}{2}$

$r_1 = \frac{-6 + 2\sqrt{7}}{2} = -3 + \sqrt{7}$, $r_2 = \frac{-6 - 2\sqrt{7}}{2} = -3 - \sqrt{7}$

$y = C_1.e^{(-3+\sqrt{7})x} + C_2.e^{(-3-\sqrt{7})x}$

11. $y'' + 9y = 0$, $y = 3$, $y' = 3$ pada $x = \frac{\pi}{3} \rightarrow y(\frac{\pi}{3}) = 3, y'(\frac{\pi}{3}) = 3$

$r^2 + 9 = 0$, $a = 1$, $b = 0$, $c = 9$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow r = \frac{-0 \pm \sqrt{0^2 - 4.1.9}}{2.1} \rightarrow r = \frac{-0 \pm \sqrt{0 - 36}}{2} \rightarrow r = \frac{0 \pm 6i}{2} = 0 \pm 3i$

$\alpha = 0 \quad \beta = 3 \rightarrow y = C_1 . e^{\alpha x} . \cos \beta x + C_2 . e^{\alpha x} . \sin \beta x \rightarrow y = C_1 e^0 . \cos 3x + C_2 . e^0 . \sin 3x$

$y = C_1 . \cos 3x + C_2 . \sin 3x \rightarrow y(\frac{\pi}{3}) = 3 \rightarrow 3 = C_1 . \cos (3 . \frac{\pi}{3}) + C_2 . \sin (3 \frac{\pi}{3}) \rightarrow$

$C_1 + 0 = 3 \rightarrow C_1 = -3$

$y' = C_1 . -\frac{1}{3} \sin 3x + C_2 . \frac{1}{3} . \cos 3x \rightarrow y' = -\frac{C_1}{3} . \sin 3x + \frac{C_2}{3} . \cos 3x$

$y'(\frac{\pi}{3}) = 3 \rightarrow 3 = -\frac{-3}{3} . \sin (3 . \frac{\pi}{3}) + \frac{C_2}{3} . \cos (3 . \frac{\pi}{3}) \rightarrow \frac{C_2}{3} . (-1) = 3 \rightarrow C_2 = -9$

$y = -3 . \cos 3x - 9 . \sin 3x$

12. $y'' + y' + y = 0$

$r^2 + r + 1 = 0$, $a = b = c = 1$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow r = \frac{-1 \pm \sqrt{1^2 - 4.1.1}}{2.1} \Rightarrow r = \frac{-1 \pm \sqrt{1-4}}{2} \rightarrow r = \frac{-1 \pm \sqrt{3i}}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$\alpha = -\frac{1}{2}$, $\beta = \frac{\sqrt{3}}{2} \rightarrow y = C_1 . e^{\alpha x} . \cos \beta x + C_2 . e^{\alpha x} . \sin \beta x$

$y = C_1 . e^{-\frac{1}{2}x} . \cos \frac{\sqrt{3}}{2} x + C_2 . e^{-\frac{1}{2}x} . \sin \frac{\sqrt{3}}{2} x$