Basic Biostatistics

Statistics for Public Health Practice

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Chapter 5: Probability Concepts

In Chapter 5:

5.1 What is Probability?

5.2 Types of Random Variables

5.3 Discrete Random Variables

5.4 Continuous Random Variables

5.5 More Rules and Properties of Probability

Definitions

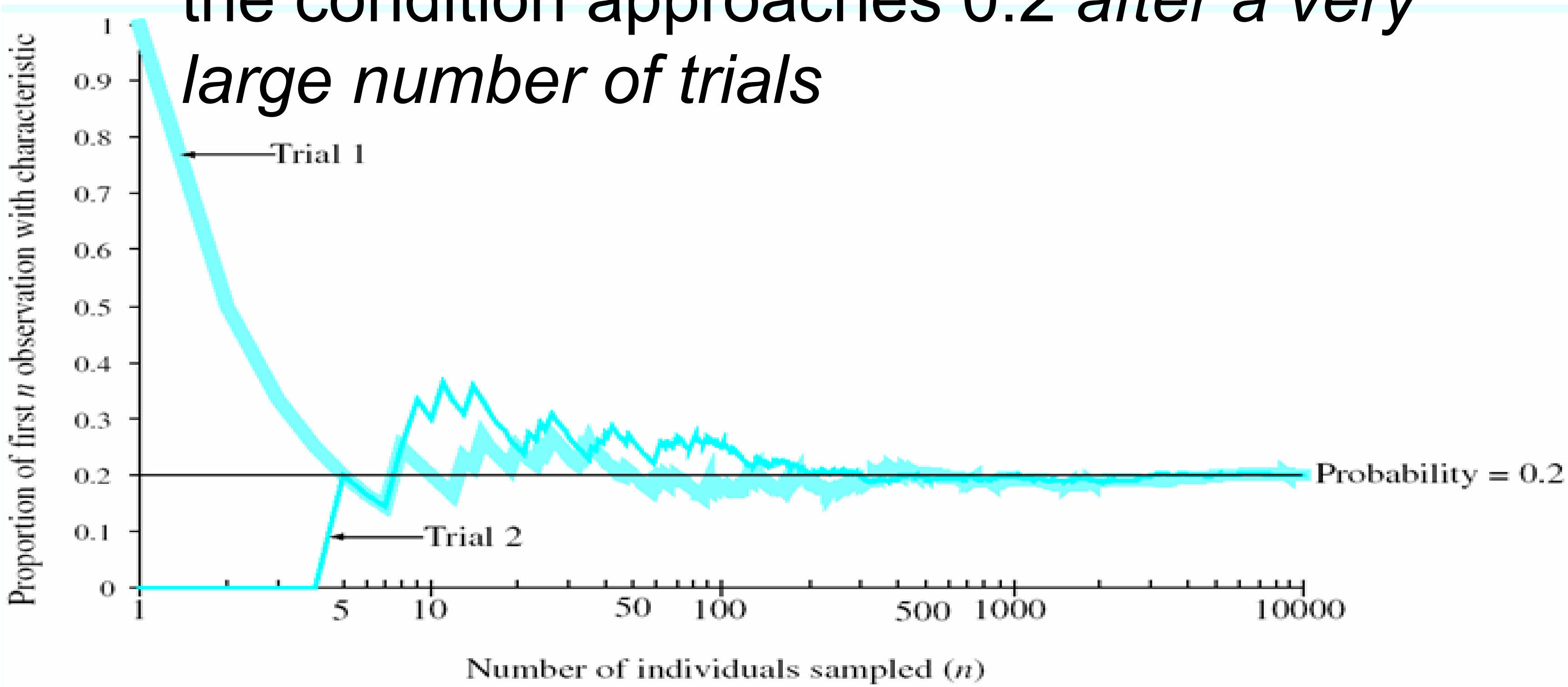
- **Random variable** ≡ a numerical quantity that takes on different values depending on chance
- **Population** ≡ the set of all possible values for a random variable
- **Event** ≡ an outcome or set of outcomes
- **Probability** ≡ the relative frequency of an event in the *population* … alternatively… the proportion of times an event is *expected* to occur in the long run

Example

- In a given year: 42,636 traffic fatalities (events) in a population of *N* = 293,655,000
- Random sample population
- Probability of event = relative freq in pop = 42,636 / 293,655,000 = .0001452 ≈ 1 in 6887

Example: Probability

- Assume, 20% of population has a condition
- *Repeatedly* sample population
- The proportion of observations positive for the condition approaches 0.2 *after a very large number of trials*



Random Variables

- **Random variable** ≡ a numerical quantity that takes on different values depending on chance
- Two types of random variables
  - **Discrete random variables** (countable set of possible outcomes)
  - **Continuous random variable** (unbroken chain of possible outcomes)

Example: Discrete Random Variable

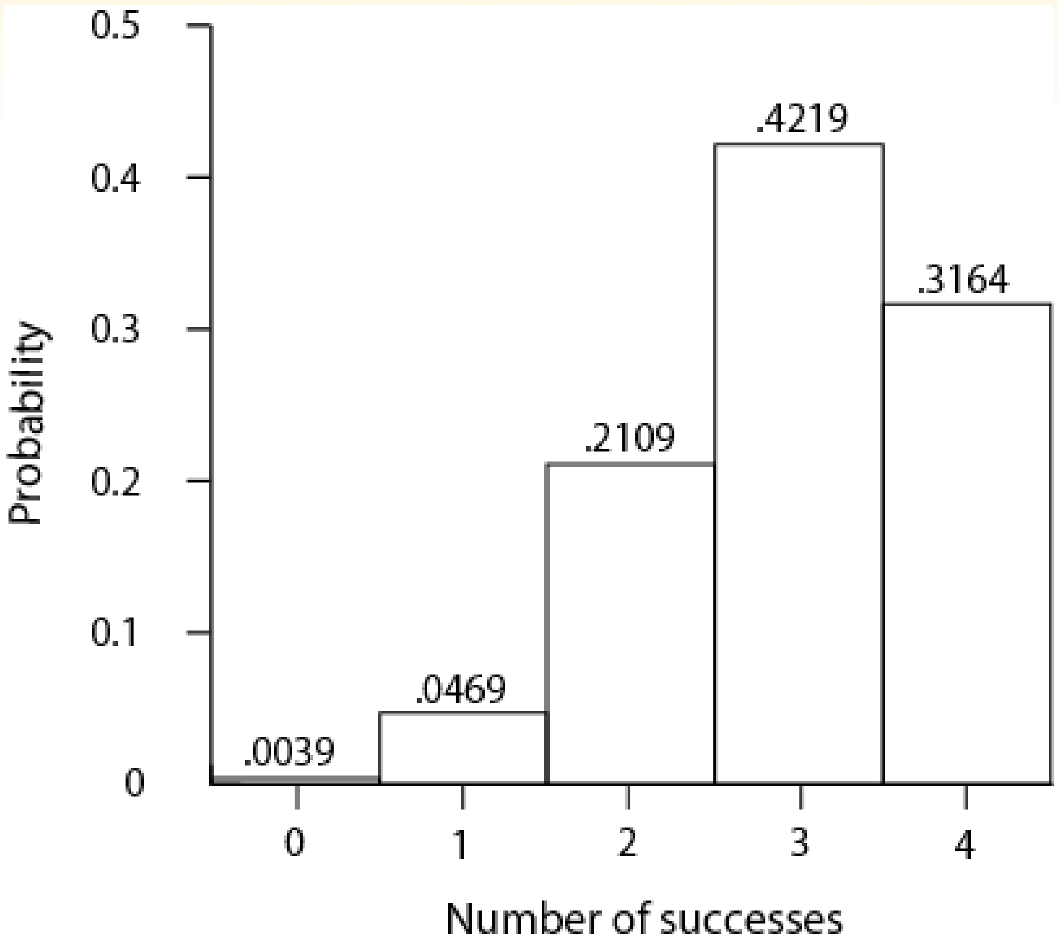
- Treat 4 patients with a drug that is 75% effective
- Let $X \equiv$ the [variable] number of patients that respond to treatment
- X is a discrete random variable can be either 0, 1, 2, 3, or 4 (a countable set of possible outcomes)

Example: Discrete Random Variable

- Discrete random variables are understood in terms of their **probability mass function (*pmf*)**
- ***pmf*** ≡ a mathematical function that assigns probabilities to all possible outcomes for a discrete random variable.
- This table shows the *pmf* for our "four patients" example:

x	0	1	2	3	4
$\Pr(X=x)$	0.0039	0.0469	0.2109	0.4219	0.3164

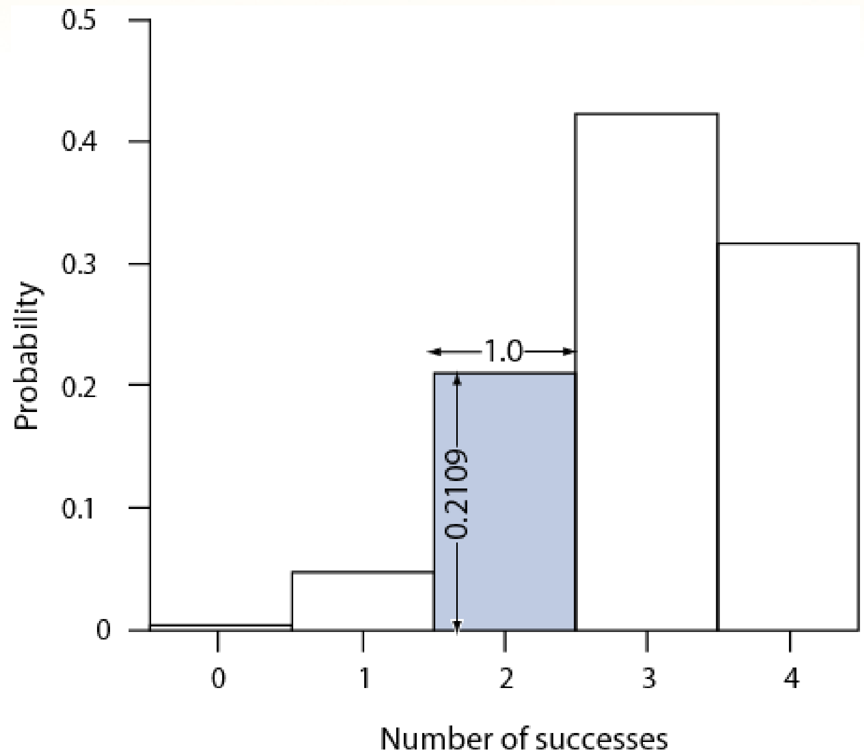
The "four patients" *pmf* can also be shown graphically



Area on *pmf* = Probability

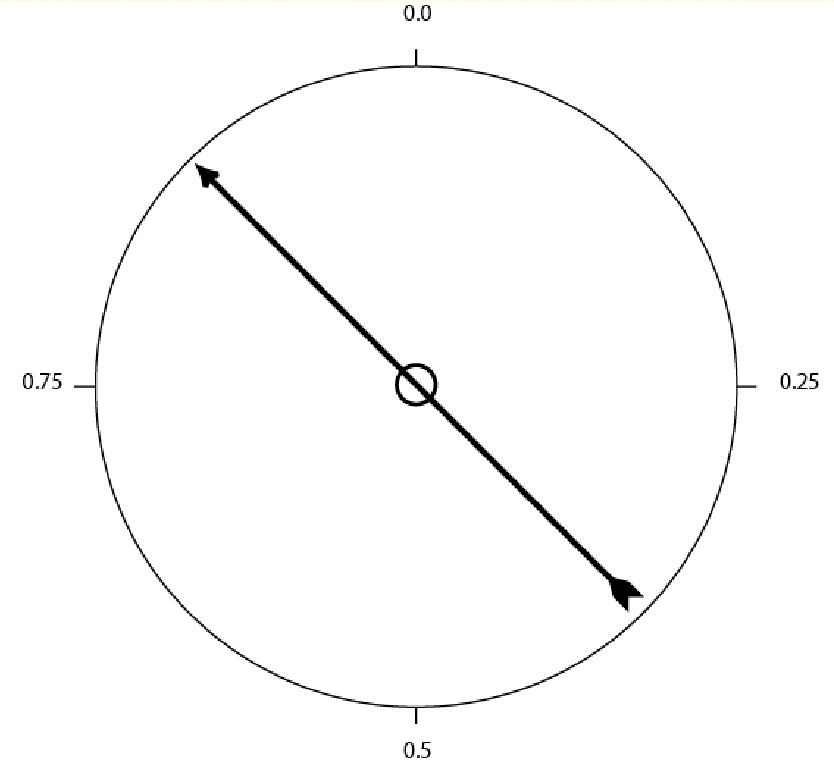
- **Areas under** ***pmf*** graphs correspond to probability
- For example:
  Pr(X = 2)
  = shaded rectangle
  = height × base
  = .2109 × 1.0
  = .2109

"Four patients" *pmf*



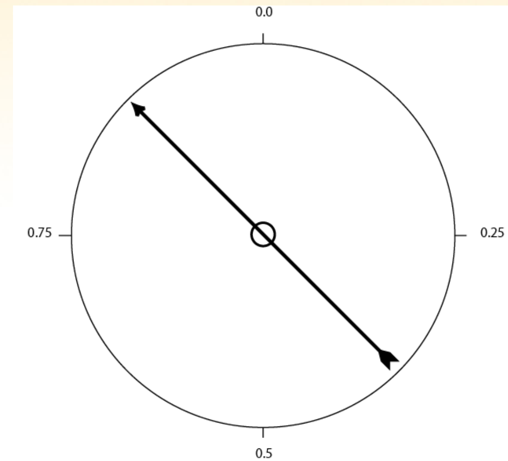
Example: Continuous Random Variable

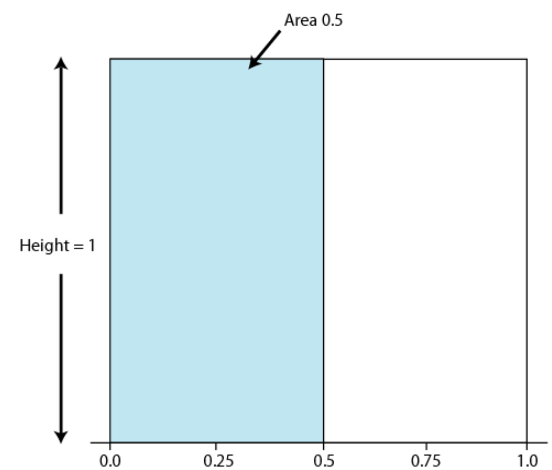
- Continuous random variables have an infinite set of possible outcomes
- **Example**: generate random numbers with this spinner ⇒
- Outcomes form a *continuum* between 0 and 1



Example
Continuous Random Variable

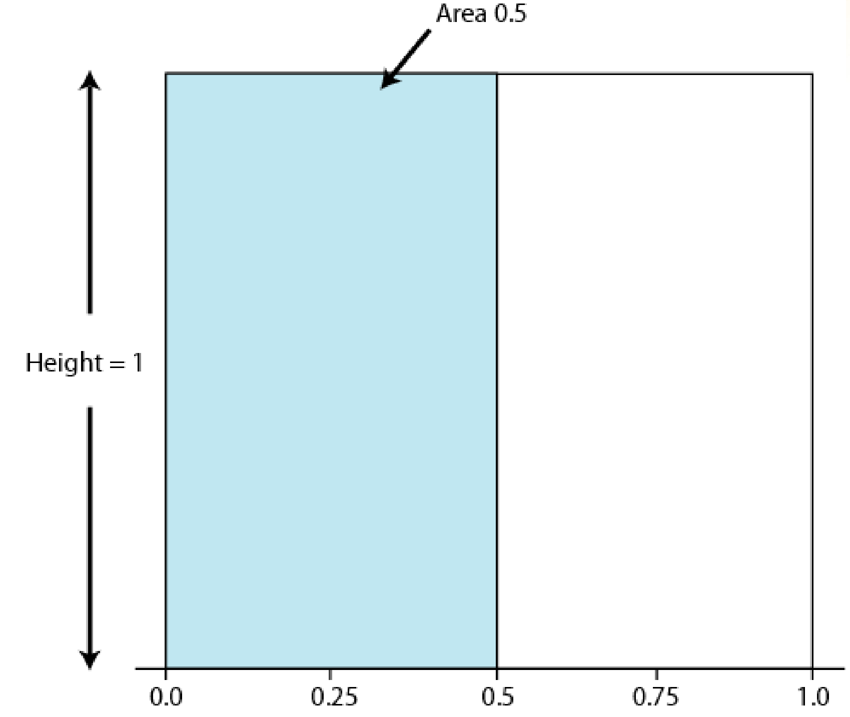
- **probability density function (*pdf*)** ≡ a mathematical function that assigns probabilities for continuous random variables
- The probability of any exact value is 0
- BUT, the probability of a **range** is the area under the *pdf* "curve" (bottom)



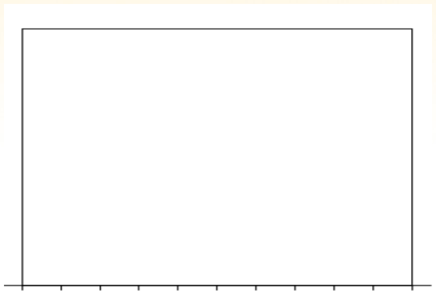


Example
Continuous Random Variable

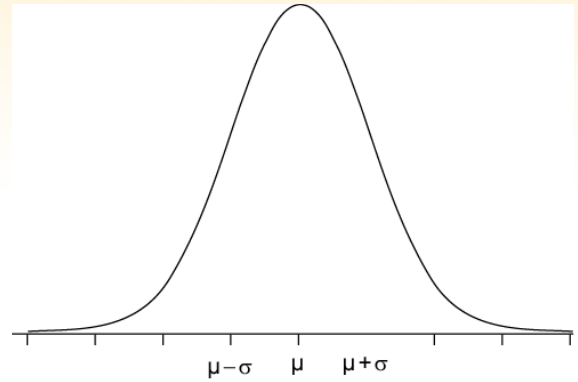
- **Area = probabilities**
- The *pdf* for the random spinner variable ⇒
- The probability of a value between 0 and 0.5 $\Pr(0 \le X \le 0.5)$ = shaded rectangle = height × base = 1 × 0.5 = 0.5



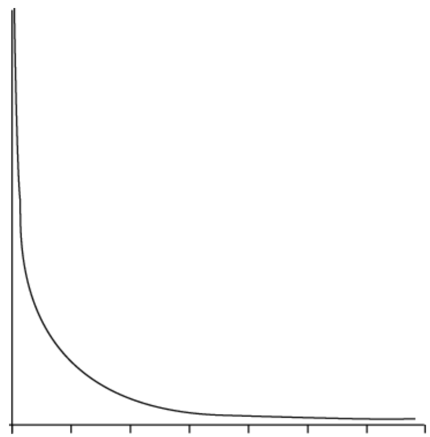
*pdf*s come in various shapes here are examples



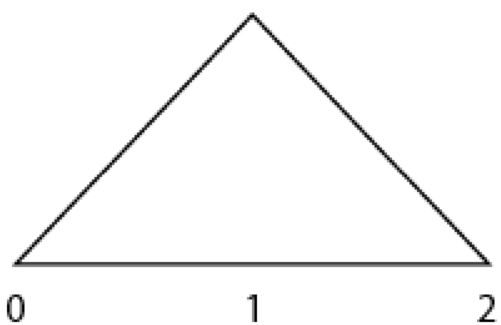
Uniform *pdf*



Normal *pdf*



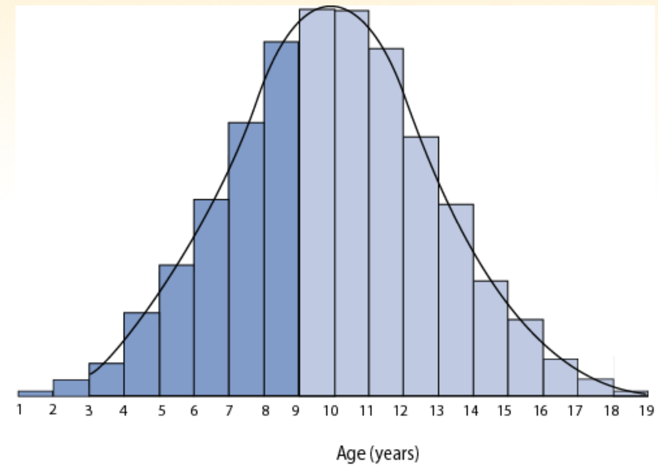
Chi-square *pdf*



Exercise 5.13 *pdf*

Areas Under the Curve

- *pdf* curves are analogous to probability histograms
- **AREAS = probabilities**
- **Top figure**: histogram, ages ≤ 9 shaded
- **Bottom figure**: *pdf*, ages ≤ 9 shaded
- Both represent proportior of population ≤ 9



Properties of Probabilities

- **Property 1.** Probabilities are always between 0 and 1
- **Property 2.** The **sample space (S)** for a random variable represents all possible outcomes and must sum to 1 exactly.
- **Property 3.** The probability of the **complement** of an event (“*NOT* the event”)= 1 MINUS the probability of the event.
- **Property 4.** Probabilities of **disjoint events** can be added.

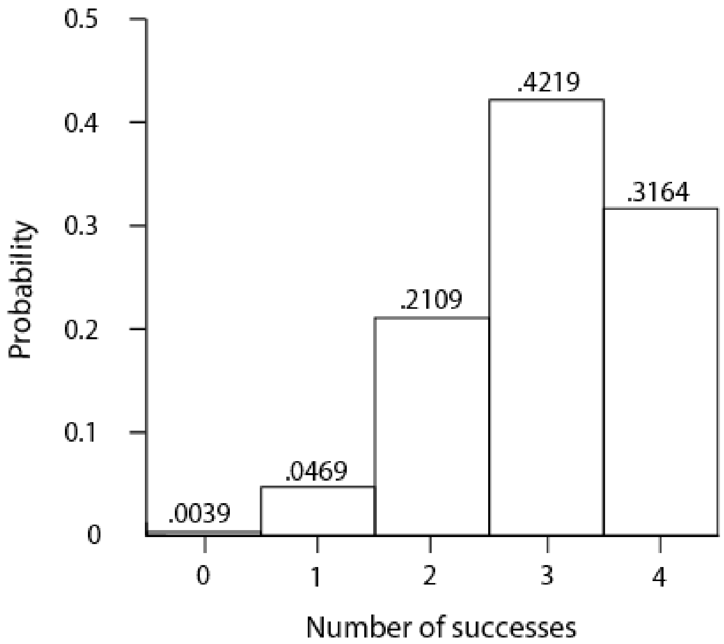
Properties of Probabilities

In symbols

- **Property 1.** $0 \le \Pr(A) \le 1$
- **Property 2.** $\Pr(S) = 1$
- **Property 3.** $\Pr(\bar{A}) = 1 - \Pr(A)$, $\bar{A}$ represents the complement of A
- **Property 4.** $\Pr(A \text{ or } B) = \Pr(A) + \Pr(B)$ when A and B are disjoint

Properties 1 & 2 Illustrated

"Four patients" *pmf*



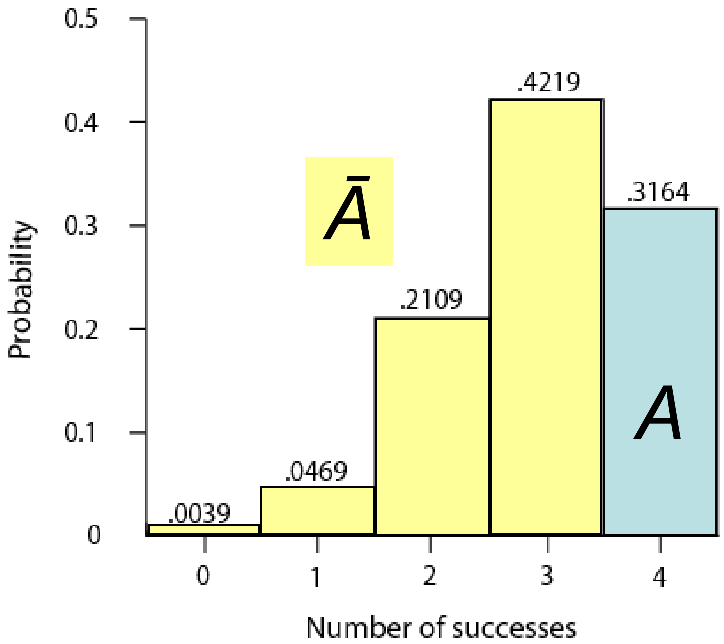
Property 1. Note that all probabilities are between 0 and 1.

Property 2. The sample space sums to 1:
Pr(S) = .0039 + .0469 + .2109 + .4219 + .3164 = 1

Property 3 ("Complements")

Let A ≡ 4 successes

"Four patients" *pmf*



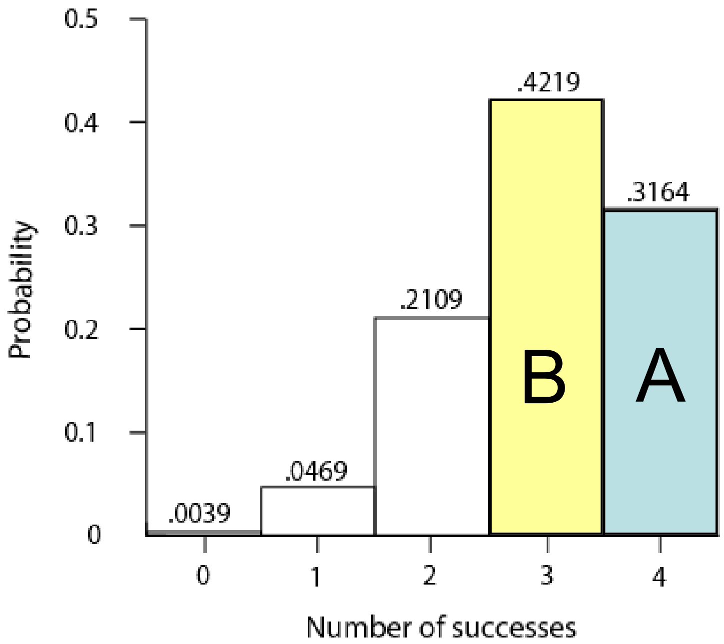
Then, $\bar{A}$ ≡ "not A" = "3 or fewer successes"

Property of complements:

$\Pr(\bar{A}) = 1 - \Pr(A)$
$= 1 - 0.3164$
$= 0.6836$

Property 4 (Disjoint Events)

"Four patients" *pmf*



Let *A* represent 4 successes

Let *B* represent 3 successes

A & B are disjoint

The probability of observing 3 *or* 4:

Pr(A or B)

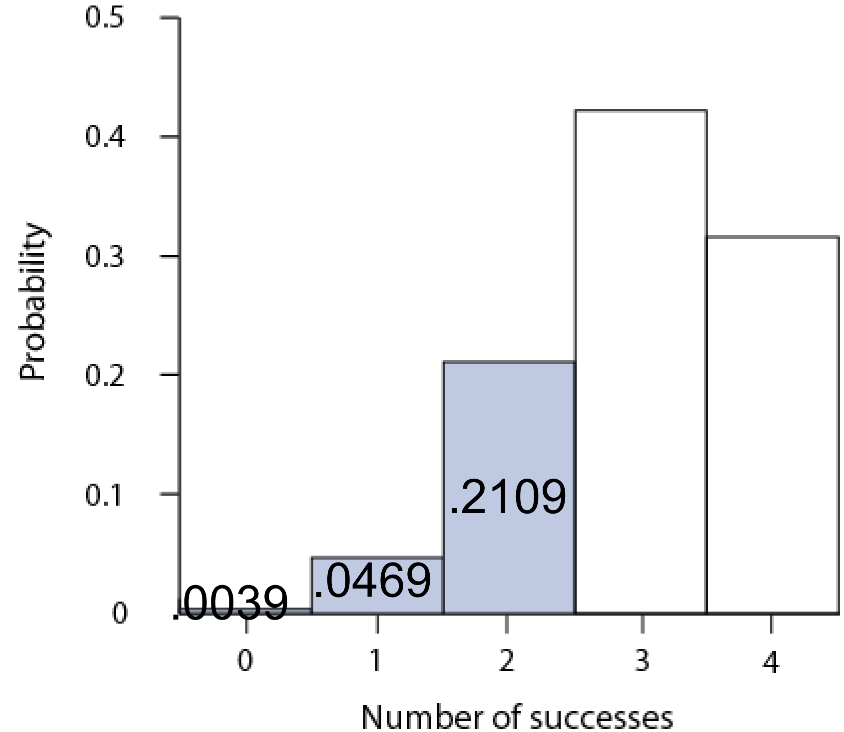
= Pr(A) + Pr(B)

= 0.3164 + 0.4129

= 0.7293

Cumulative Probability Left "tail"

- ***Cumulative*** *probability* = probability of x *or less*
- Denoted **Pr(*X* ≤ *x*)**
- Corresponds to area in **left tail**
- Example:
  Pr(X ≤ 2)
  = area in left tail
  = .0039 + .0469 + .2109
  = 0.2617



Right *"tail"*

- Probabilities greater than a value are denoted **Pr(*X* > *x*)**
- Complement of cumulative probability
- Corresponds to area in **right tail of distribution**
- Example (4 patients *pmf*):
  Pr (X > 3)
  = complement of Pr(X ≤ 2)
  = 1 - 0.2617
  = .7389