# Nonlinear Programming Problem of Bandwidth Based and VolumeBased Call Pricing Scheme in Wireless Networks

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#### Abstract

The pricing for wireless networks is developed to obtain surplus from subscribers. The linearity factors, elasticity price, price factors are discussed. The new approach of wireless pricing model proposed by previous research are approached by considering the model as the nonlinear programming problem that can be solved optimally using LINGO 13.0. The problem is considered to be nonlinear programming that can be solved using optimization tools. The solutions are expected to give some information about the connections between the acceptance factor and the price. The models attempt to maximize the total price for a connection based on Quality of Service (QoS) parameter. The maximum goal to maximum price is achieved when the provider set the increment of price change due to QoS change and amount of QoS value. The linearity parameter set up for most cases is obtained in ceiling value. Linear price factor ranges between the prescribed value especially cases when we increase the price change due to QoS change and increase the amount of QoS values.

**Keywords:** Wireless networks, Maximize total price, QoS parameter

#### 1 INTRODUCTION

The work that survey on various pricing schemes in networks beginning with the flat rate scheme such as the fixed price plan, time based plan, number of packets plan, single fee plan, Paris Metro Pricing, responsive pricing and reservation based. Other categories are parameter based such as static pricing, smart market pricing, priority pricing, adaptive pricing, cumulus pricing, dynamic pricing, assignment pricing, rate adaption pricing and programmable pricing [1, 2]. Others also discussed some optimization problem of pricing scheme based on bandwidth parameter [3-5] and solved iteratively using LINGO.

The development of wireless networks plays critical role in business life. The issue of optimal pricing that is the condition when the provider has the rights to maximize the revenue

[6]. The beginning research focus on pricing in networks due to Responsive Pricing [7], by performing three stages proposed consist of not using feedback and user adaptation, using the closed-loop feedback and one variation of closed loop form. The pricing plan proposed by Altmann [8] was comprised of the flat rate and usage based pricing. Proposed pricing scheme offers the user a choice of flat rate basic service, which provides access to internet at higher QoS, and ISPs can reduce their peak load. Pricing strategy proposed by Byun [9] is based on economic criteria. They Design proper pricing schemes with quality index yields simple but dynamic formulas. Changes in service pricing can be created and so the revenue. Optimal pricing strategy proposed by Wu [10] is designed by setting up the Flat fee, Pure usage based, Two part tariff. Supplier obtains better profit if chooses one pricing scheme and their ability to pay. Paris Metro Pricing [11, 12] stated that different service class will have a different price. The scheme makes use of user partition and move to choose other class it found same service from other class with lower unit price.

Class of QoS represents the resource reservation but we do not always use the resource. Therefore, by using QoS class, we can implement QoS based pricing for wireless networks [13, 14]. Some recent works describe LTE network pricing scheme that utilizes the QoS parameter, user sensitivity resource block and classify the class into three classes namely gold, silver and bronze [15] and by classifying this, the provider can evaluate the models that suit to the goals of their project.

The pricing models in some part do not really mention about the availability for QoS differentiation and some works does not create the optimization problem to approach the solutions. Pricing for 3G network [13], states that the linearity factor, acceptance factor, elasticity price, traffic load, the provider able to maximize the price and maximize the call function, the solution can be achieved. Pricing strategy proposed by Safari [16] was designed by considering the optimal pricing strategy for specific service as function of time. Their proposed model was created then comparing with the existing approaches available. The models focus on continuous models solved heuristically. Meanwhile, the pricing strategy [17] stated that the dynamic pricing scheme proposed by setting up the model as a partial differential equation (PDE) and solving it numerically. The pricing scheme proposed mainly for pricing companies. Their work utilizes the PDE background by utilizing necessary and sufficient condition of Lagrange. So by solving the boundary conditions the pricing scheme involving company debt can be calculated. Simulation method for designing network proposed by Kennington [18] is able to examine the schemes that are not reached by network testing and able to improve model and performance. Concept of Dynamic pricing introduced by Smyk [19] explains that process to fluctuate prices between consumer and provider. In market condition, the re-priced can often occur. Also, pricingQoS strategy proposed by Jang [20] explains the utility function and cost function are proposed, and pricing mechanism is based on QoS service classes. Strategy proposed by Maill [21] was chosen by creating a model, the discussion on fixed prices, demand is giving to providers, price war is categorized by using the Nash equilibria.

Though some works show the models of wireless pricing based QoS but the discussions about formulating the models into optimization problem are seldom discussed. The main advantage in transforming into mathematical programming is the reasonable models can be designed to support the decision and the decisions are needed in business life. So, in this paper, the new approach of wireless pricing model proposed by [13, 14, 22] are approached by

considering the model as the nonlinear programming problem that can be solved optimally using LINGO 13.0. Two main call function schemes, bandwidth based and volume based are discussed along with the three QoS attributes to see the performance of those three QoS attributes.

## 2 RESEARCH METHODOLOGY

The models designed are adapted from [13, 14, 22] but the new approach is the by transforming into mathematical programming known as optimization problem approach. The models then solved iteratively using LINGO 13.0. Basically, the models attempt to maximize the total price for a connection based on QoS parameter. The total price is the summation between basic price for a connection and the price change due to QoS change. We have i users and j class. The models are based on two set of categories. First by bandwidth based scheme and second one is volume based scheme. Then, we can examine for each case, which scheme offer the best solution.

The objective of the research is to obtain the revenue for the provider. The model provided by [13, 14] is available. But here, we create the models by gathering all information about parameter and variables and transform all into mathematical programming problem.

So, the objective function will maximize

$$\sum_{j}^{m} \sum_{i}^{n} (PR_{ij} + PQ_{ij}) \tag{1}$$

which means to maximize the summation of total price that consists of the price for a connection with QoS available and the price change over that QoS. The objective function has limitation to be satisfied to obtain the revenue which is called the sets of the constraints.

The first constraint states that the price change will depends on the factor of the price, that involves the bandwidth as QoS attribute, the basic price at user i and class j, and also the factor of linearity. Then, we have

$$PQ_{ij} = (1 + \frac{x}{2000})PB_{ij}L_x, for Bandwidth QoS attribute$$
 (2)

$$PQ_{ij} = (1 + \frac{x}{350})PB_{ij}L_x$$
, for End – to End Delay QoS attribute (3)

Where PBij is the basic price for a connection for user i and the class j and Lx is the linearity factor. Then, aijwhich defines the linear price factor in user i and class j, the linear factor  $(e - e^{-Bx})$  and the traffic load tl. So,

$$PB_{ij} = \frac{\alpha_{ij}((e - e^{-Bx}))t_l}{100} \text{ for bandwidth based continuous call pricing scheme, or}$$
 (4)

$$PB_{ij} = \alpha_{ij}(e - e^{-Bx})$$
 for Volume based continuouscall pricing scheme, or (5)

Lx is a linearity factor that depends on the linearity parameters of a and  $(e - e^{-Bx})$ . Then

$$L_x = \alpha(e - e^{-Bx}) \tag{6}$$

With x is assumed between 0 and 1.

The linear price factor aijis set up between prescribed values determined by the provide., say f and g. So,

$$f \le \alpha_{ij} \le g \tag{7}$$

The range of allowed traffic load  $t_{-}$ l is also determined by the providers, say h and k. Then,

$$h \le t_l \le k \tag{8}$$

For x as the amount of increment of decrement in QoS value, we range between 0 and 1 implying 0 is in best effort service case while 1 means in perfect service case. B is arranged between 0.8 and 1.07 since in this range, the best network quality occurs.

$$0 \le x \le 1 \tag{9}$$

$$0.8 \le x \le 1.07 \tag{10}$$

For parameter value PRij, the provider arranges the value to have a connection. It also happens in a as the linearity parameters that keep the ratio of the price between floor and ceiling of QoS value is not really high.

Next step, for a model described above, we solve to obtain the optimal solution for each case involving the 3 QoS attributes for bandwidth based and volume based continuous call pricing schemes. Table 1 and Table 2 summarize the solver status for all cases, respectively.

Table 1: Solver Status for Bandwidth Based Continuous Call Pricing Scheme.

QoS Attribute	Bandwidth	BER	End-to-End Delay
Model Class	Nonlinear Programming	Nonlinear Programming	Nonlinear Programming
State	Local Optimal	Local Optimal	Local Optimal
Objective	32.6816	32.7534	$3.0468 \times 107$
Infeasibility	0	$8.8817 \times 10\text{-}16$	$1.8626 \times 10-9$
Iterations	18	18	19
*GMU	25	25	25
**ER	0	1	0

<sup>\*</sup>GMU stands for Generated

Memory Used

Run time

<sup>\*</sup>ER stands for Elapsed

## 3 RESULTS

Table 1 and Table 2 depict the results for each scheme of continuous call pricing schemes. In Table 1 and Table 2, model class for each QoS attribute defined as nonlinear programming, having local optimal state. The highest objective value to maximize the price for each user is achieved when setting up the QoS attribute of End-to-end delay. In addition, when the scheme is bandwidth based, the objective function value reach the highest value than in volume based scheme since in volume based, the traffic load is already set up as 1. In some cases of QoS attribute, we find the a very small value of infeasibility of the model to show amount constraints are violated, which is a very small value, or we can say, tends to zero. Also, The Extended Solver Status box details similar information for the more advanced branch-and-bound, we have number of iterations required to solve the model which are basically from 24-25 iterations to achieve the optimal solutions.

Table 2:	Solver	Status	for	Volume .	Based	Continuous	Call	Pricing Schei	me

QoS Attribute	Bandwidth	BER	End-to-End Delay
Model Class	Nonlinear Programming	Nonlinear Programming	Nonlinear Programming
State	Local Optimal	Local Optimal	Local Optimal
Objective	5.24816	5.25534	$3.04664 \times 106$
Infeasibility	0	0	0
Iterations	16	16	14
*GMU	24	24	25
**ER	0	0	1

## 4 CONCLUSSIONS

The maximum goal to maximum price is achieved when applying the bandwidth based call pricing scheme, since the traffic load takes into considerations. For bandwidth and bit rate error QoS attributes, the difference in objective function value is quite high rather than in end-to-end delay QoS attribute.

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