A Logistics Networks Optimisation Model

for Warehouse Operation Selection Problem

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*Abstract*— One important strategic issue related to the design and operation of a physical distribution network is the determination of the best sites for warehouses for intermediate stocking points as well as its capacity. Therefore, it is important to integrate the facility location models and decision with logistical functions and components in the logistics performance analysis. This paper focuses on optimizing the capacitated multi-items, multi-facilities, multi-periods logistics network problem. It differs from previous research in that the model includes the selection of warehouse location and capacity, and also considers backlogging. A mixed integer linear programming (MILP) model is developed. To validate the model, several small data sets are used. The problem is solved using commercial linear programming package, CPLEX ver. 11. From the experiments there are several managerial insights. First, networks with higher warehouse capacity can expect better reduction in total network cost. Second, the number of warehouses activated and its capacity selected are related to the total demand that must be satisfied. Third, networks with high fixed setup cost tend to increase the production level to maximum capacity and store more inventories distributed in several periods And, networks with allowing backorder have a better performance when handling low demand and high demand.

Keywords—Logistics Network Optimisation, Mixed Integer Linear Programming , Backlogging, Capacity Selection

# Introduction

In industry, one important strategic issue related to the design and operation of a physical distribution network is the determination of the best sites for warehouses for intermediate stocking points as well as its capacity. The use of warehouses provides a company with flexibility to respond the changes in the marketplace and can result in significant cost savings due to economies of scale in transportation or shipping costs. Therefore, it is important to integrate the facility location models and decision with logistical functions and components in the logistics performance analysis.

In order to improve the logistics network performance in an integrated way, we need to consider the integration of production planning, and the distribution problems. This includes the merging of the production, inventory, and transportation problems in a single formulation. This is very crucial in logistics system optimization. In a company operation, a production planner is concerns with optimizing the production and inventory level in each period so that the cost is minimized. On the other hand, the distribution planner is concerns with determining the distribution plan to satisfy customer demand so that the transportation cost is minimized. Without the integrated analysis, these two processes independently could increase the inventory holding cost and yield longer lead times. Therefore, a company needs to explore a better model in order to achieve its objective of minimizing the total costs.

The integration of production planning and distribution model systems needs to be developed in a strategic perspective, tactical as well as operational [1]. Strategic decisions are generally long-term in scope. These decisions include the number, location and capacities of plants and warehouses, or the flow of material through the logistics network. Tactical decisions have a time horizon of several months up to one year. These decisions include purchasing and production decisions, inventory policies and transportation strategies. The operational decisions involve the day-to-day activities of a business operation such as scheduling, and vehicle routing. The operations of production and distribution can be decoupled if there is a sufficient amount of inventory between them [2].

The importance of the integrated analysis in logistics network problems have been considered by several authors and substantial evidence exists in the real business application ([3], [4], [5]) to demonstrate that integrating decisions can lead to substantial increases in efficiency and effectiveness. Therefore, the crucial questions that arise in business application nowadays are: how can we develop an integrated model from which we are able to improve the logistics network performance of a company especially in the problem of the capacity selection of company facilities.

This paper focuses on optimizing the capacitated multi-items, multi-facilities, multi-periods logistics network problem. It differs from previous research in that this paper includes the selection of warehouse location and capacity, and we also consider backlogging. The problem is optimizing the production and distribution plan over a finite time horizon to satisfy demand requirements while determining the best selection of warehouses to be used from available warehouses and choosing its optimum capacity and the best strategy to distribute products to customers.

# Literature Review

Over the past several decades, many authors have proposed models and methods for solving the facility location problem. Francis, McGinnis, and White [6] provided a review on formulation and solution of facility location problem. Aiken [7] provided a review on facility location problem on physical distribution management. Current, Min, and Schilling [8] provided a review on model of multi-objectives facility location problems. Sridharan [9] provided an extensive review of capacitated plant location problems. Owen and Daskin [10] reviewed facility location model in strategic planning decisions.

There are many classifications of location problem that have been widely studied. One is the uncapacitated facility location problem versus capacitated problem. The uncapacitated problem was studied, among others, by [11], [12], [13], [14], [15] and [16]. Resende and Werneck [17] presented a multi-starts heuristic for the uncapacitated facility location problem. The capacitated facility location problem was studied, among others, by [18], [19], [20], [21], [22], and [23]. Klose and Drexl [24] provided a review on continuous location models, network location models, mixed integer programming models, and their application. Wu, Zhang and Zhang [25] also provided an extension of capacitated facility location problem in which the general setup cost functions and multiple facilities in one site are considered.

The integration of facility location and logistics aspects had also been studied by a number of authors, such as [26], [27], [28], and [29]. Jayaraman and Pirkul [26] provided a study on integrated logistics model for locating production and distribution facilities in a multi-echelons environment. They model both strategic and operational decisions to design and test a production and distribution system model and evaluate its performance.

Syam [27] stated that the critical logistics issue in current logistics problems is the determination of optimal locations for plants and warehouses, as well as the determination of optimal consolidation policies, given the set of open sites. He provided a location-consolidation model that simultaneously determines facility locations, flows, shipment compositions, and shipment cycle times in a multi-commodities, multiple plants and warehouse environment.

Melo, Nickel, and Gama [28] provides a mixed integer linear programming model for dynamic facility location that captures important features of strategic supply chain planning problems. The features include the relocation of existing facilities through capacity transfer to new location, integration of inventory, transportation, and supply decision, the availability budget for investment in facility location and relocation and supply decision, the generic structure of the supply chain network.

Thanh, Bostel and Peton [29] proposed a dynamic model for facility location and supply chain design. Their model is a mixed integer linear programming model for a multi-commodities multi-echelons production-distribution network with deterministic demand. The features are selection of suppliers, opening or closing facilities, planning capacity for existing facilities, production management and distribution management as well as inventory management.

# Multi-Item, Multi-Facilities Network Problem Formulation

We consider the problem of designing a distribution network that involves determining simultaneously the best selection of warehouses used and the best strategy for distributing the product from the plants to the warehouses and from the warehouses to the customers. Our problem has the following features:

* *l* plants P1, P2, …, Pl where the product can be produced.
* *m* potential warehouses sites W1, W2, …, Wm where the product can be stored.
* *q* products identified as 1,2,…q, which can be produced in the plants over a period of time. In general, it is assumed that in each time period, the production facility is allowed to produce more than one product, coupled with limited production capacity.
* *r* capacity level available identified as 1,2,…, R of the potential warehouses sites
* *n* customer locations C1, C2, …, Cn where the product is required.
* Plant production capacities for each product and storage capacities of the potential warehouses are known.
* Customer requirements at each centre are deterministic and known for each period. Furthermore, backorders are allowed and permitted.
* All products are assumed delivered directly to warehouse.
* A planning horizon of T periods.
* A homogeneous fleet of vehicles to transport the product through the network.

## Model Parameters

For each product *q* there are following notation for the cost data: : setup cost; : unit cost of production; : unit cost of transportation to deliver product from plant to warehouse*;* : unit cost of transportation to deliver product from warehouse to customer*;* : unit inventory holding cost at warehouse*;*  : unit backorder cost at warehouse*;* and : fixed cost of operating a warehouse*.*

The other notations that we use are: for capacity of plant, for capacity of potential warehouse, for demand for product, for inventory level at warehouse at the end of period, for backorder level of product at warehouse, for product inventory level that is carried over from the previous period at warehouse.

## Decision Variables

The decision variables that we use are: represents the amount of product *q* produced at plant *i* in period t; represents the amount of product *q* transported from plant *i* to warehouse *j* in period t; represents the amount of product *q* transported from warehouse *j* to customer *k* in period t; represents the binary setup variable at plant *i* inproducing product *q* in period t; and represents the binary variable at warehouse *j* with capacity level *r* in period t.

## Problem Formulation

The problem is to minimize the total cost of production, transportation, and inventory over the T periods. The model assumes no starting inventory. The studied can be formulated as a MILP as follows:

Minimize

where (1)

and

Subject to following constraints:

(2)

(3)

(4)

(5)

(6)

(7)

(8)

(9)

(10)

(11)

(12)

(13)

(14)

(15)

(16)

= 0 or 1 (17)

= 0 or 1 (18)

= 0 or 1. (19)

# Computational Results and Discussions

## Data

In this section we describe the computational experience in solving our model. We generate several test data to demonstrate the applicability of the mathematical formulations above. The following data are used:

* There are 9 nodes which represent 2 Plants, 5 Warehouses, and 2 Customers.
* T period observation is 12 period.
* Plants production capacities as well as inventory storage are a specific range of units/period.
* Demand model are deterministic.
* Setup cost model, production cost model in production facilities, and transportation cost between plants, warehouses, and customers as well as costs incurred in warehouses are specified.

In order to test and illustrate the impact of different factor on proposed model, 40 test problems are generated which are categorized in 10 cases. For each case we generate 4 scenarios of various setup costs and demand variation model. There are two demand models which are low level demand and high level demand, and two types of setup cost (low and high setup cost).

The optimal production schedule and product movements as well as its total costs are computed for each configuration under all scenarios using CPLEX ver 11. The experiments were conducted on personal computer with Intel Core 2 Duo 2.66 GHz and 2 Gbyte memory.

## Results and Discussions

The results of this section seek to illustrate the effect of increasing various model costs on the warehouses and its capacity selection. There are three warehouse capacities which should be selected We observe using two plants capacity option: low capacity and high capacity.

First, we examined the case if we use low plants capacity. We examined how the total network cost change as we select the available warehouses and its capacity in storing products. We change the setup cost, from low cost to high setup cost, and vary demand from low to high demand setting. We classify the result based on the various cases. We found that the total network cost increases as we increase the setup cost, except in case 1 where the result is infeasible. We observed that having a high setup cost when handling demand changing from low demand to high demand generally is more favorable in low plants capacity model as resulting lower average total cost increment, 19.31% compare to 19.71% if we use lower setup value.

We observed that the optimal number warehouses should be operated is 3 warehouses. When we use 3 warehouses we gain total cost reduction of: 4.14% in scenario A; 4.20% in scenario B; 3.71% in scenario C; and 3.74% in scenario D. While using 4 or 5 warehouses there are no further reduction obtained. We found that if we only use 1 warehouse to supply demand, the result is infeasible due to the insufficient capacity in our network. This finding brings us to the fact that our optimal solution will be achieved when we have sufficient capacity, in this case the warehouse capacity.

We observed the optimal warehouses schedule and found that when demand is low our model tend to operate warehouse type 1 and 2, except in case 2 where we limit to operate maximum only 2 warehouses (Table 1). In case 2, all types of warehouses are used. If demand is high, all types of warehouses will be used. From our results, we observed that the most warehouses used are warehouse 3 and 4.

From Table 1 we also observed that by allowing backorder in the model, we gain cost reductions as we compare to the result of no-backorder model. We conclude that by allowing backorder in our model, we can handle demands better than basic model.

1. Comparison of total network cost reduction between backorder case and no-backorder case

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| No of Warehouse | Scenario A | Scenario B | Scenario C | Scenario D |
| 2 | -0.19% | -0.26% | -0.21% | -0.30% |
| 3 | -0.04% | -0.09% | -0.03% | -0.02% |
| 4 | -0.04% | -0.09% | -0.03% | -0.02% |
| 5 | -0.04% | -0.09% | -0.03% | -0.02% |

Another observation from this case problem could be seen in Table 2. We summarize the running time of all scenarios and find that the problem difficulty to solve depends on the maximum number of warehouses operated and the values of the fixed costs. When we only allowed operating maximum 2 warehouses, our problems are more difficult to be solved as indicated in higher solution times. In other cases, the scenarios are solved faster. Table 2 is a result of the mixed integer linear programming relaxation done in CPLEX with emphasis on the optimality. We set a dynamic search as a searching method.

1. CPLEX running time of case with low plants capacity

|  |  |  |  |
| --- | --- | --- | --- |
| Case Scenario | Solution Time (Sec) | Case Scenario | Solution Time (Sec) |
| LP-2W-A | 20.53 | LP-4W-A | 0.25 |
| LP-2W-B | 167.31 | LP-4W-B | 0.91 |
| LP-2W-C | 60.31 | LP-4W-C | 1.19 |
| LP-2W-D | 288.31 | LP-4W-D | 2.94 |
| LP-3W-A | 0.19 | LP-5W-A | 0.19 |
| LP-3W-B | 2.50 | LP-5W-B | 0.78 |
| LP-3W-C | 0.45 | LP-5W-C | 0.53 |
| LP-3W-D | 2.94 | LP-5W-D | 3.16 |

Finally, we examined the case if we use high plants capacity. We examined how the total network cost change as we select the available warehouses and its capacity in storing products. We change the setup cost, from low cost to high setup cost, and vary demand from low to high demand setting. We found that the total network cost increases as we increase the setup cost, except in case 6 where the result is infeasible. We observed that having a high setup cost when handling demand changing from low demand to high demand generally is more favorable in low plants capacity model as resulting lower average total cost increment, 17.69% compare to 17.92% if we use lower setup value.

We observed the effect of number warehouses operated to network total costs. We found that if we only use 1 warehouse to supply demand, the result is infeasible due to the insufficient capacity in our network. This finding brings us to the fact that our optimal solution will be achieved when we have sufficient capacity, in this case the warehouse capacity. It can be referred to constraint (5) in our model that indicates the property of our optimal solution.

We observed that the optimal number warehouses should be operated is 3 warehouses. When we use 3 warehouses we gain total cost reduction of: 4.55% in scenario A; 4.57% in scenario B; 4.00% in scenario C; and 4.05% in scenario D. While using 4 or 5 warehouses there are no further reduction obtained. We also observed that by allowing backorder in the model, when we supply high demand we gain cost reductions as we compare to the result of no-backorder model. However, no further cost reduction if we cover the low demand, except in scenario B (Table 3). We can conclude that by allowing backorder in our model, we can handle demands better than basic model.

1. Comparison of total network cost reduction between backorder case and no-backorder case

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Number of Warehouse | Scenario A | Scenario B | Scenario C | Scenario D |
| 2 | 0.00% | -0.02% | 0.00% | -0.01% |
| 3 | 0.00% | 0.00% | -0.02% | -0.05% |
| 4 | 0.00% | 0.00% | -0.02% | -0.05% |
| 5 | 0.00% | 0.00% | -0.02% | -0.05% |

We also observed the optimal warehouses schedule and found that in all cases and scenarios warehouse type 1, type 2 and type 3 will be used.

1. CPLEX running time of case with high plants capacity

|  |  |  |  |
| --- | --- | --- | --- |
| Case Scenario | Solution Time (Sec) | Case Scenario | Solution Time (Sec) |
| HP-2W-A | 0.31 | HP-4W-A | 0.06 |
| HP-2W-B | 0.38 | HP-4W-B | 0.08 |
| HP-2W-C | 0.14 | HP-4W-C | 0.08 |
| HP-2W-D | 0.31 | HP-4W-D | 0.09 |
| HP-3W-A | 0.06 | HP-5W-A | 0.05 |
| HP-3W-B | 0.08 | HP-5W-B | 0.05 |
| HP-3W-C | 0.06 | HP-5W-C | 0.06 |
| HP-3W-D | 0.08 | HP-5W-D | 0.06 |

Another observation from this case problem could be seen in Table 4. We summarize the running time of all scenarios and find that the problem difficulty to solve depends on the maximum number of warehouses operated and the values of the fixed costs. When we only allowed operating maximum 2 warehouses, our problems are more difficult to be solved as indicated in higher solution times. In other cases, the scenarios are solved faster. Table 4 is a result of the mixed integer linear programming relaxation done in CPLEX with emphasis on the optimality. We set a dynamic search as a searching method.

# Conclusion

In this paper we study a class of production-distribution problems arising in logistics supply chain networks. We discussed a facility selection problem in logistics network. In particular the selection of warehouses activation and its capacity selection. We studied the capacitated multi-items, multi-facilities, multi-periods logistics network problem. Our problem considers fixed-charge production cost with production capacities, fixed-charge warehousing cost with warehousing capacities, and linear transportation cost. We also considered the problems with transportation capacities and backorder capability. The models that we studied help managers to answer question that arise in managing the logistics network.

In order to study our problems, for each model we generated several test problems which are categorized into 10 cases and 4 scenarios. In each case, we carried out our experiments by varying the plant’s capacity and the number of warehouses operated. To illustrate the impact of different factors, we generate our scenarios by varying the setup cost, and demand models. The problems are solved using CPLEX version 11. Our results indicate that the performance of our model depends on the value of setup costs, the value of demands, and the number of warehouses activated as well as the value of capacity of each facility.

From those results we obtain several managerial insights. First, networks with higher warehouse capacity can expect better reduction in total network cost. Second, the number of warehouses activated and its capacity selected are related to the total demand that must be satisfied. Third, networks with high fixed setup cost tend to increase the production level to maximum capacity and store more inventories distributed in several periods. Fourth, the network configuration will avoid storing inventories in warehouse with high holding cost. Fifth, the capacity of transportation line directly affects the total network cost. It is increased when its capacity tightens. Finally, networks with allowing backorder have a better performance when handling low demand and high demand.

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