Mixed Integer Linear Programming Models for Improving the Performance of Logistics Networks

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Abstract

The role of logistics nowadays is expanding from just providing transportation and warehousing to offering total integrated logistics. To remain competitive in the global market environment, business enterprises need to improve their logistics operations performance. Improvements can be achieved through a comprehensive analysis and optimisation of the logistics networks performance. In this paper, a mixed integer linear programming model for optimizing logistics network performance is developed. We consider a single-product, multi-period, multi-facilities network, as well as the multi-product network. The problem is modeled in form of a network flow problem with the main objective being to minimize total logistics cost. The problem is solved using commercial linear programming package, CPLEX ver. 9.0.

Keywords: Integrated logistics network, Mixed integer programming, Network optimization

1. Introduction

Logistics deals with the planning and control of material flows as well as related information in organizations, and becomes a critical part of supply chain management. Its mission is to get the right materials to the right place at the right time [15]. Logistics also deals with mobility concepts relating to tangible as well as intangible assets.

Logistics activities connect and activate the objects in the supply chain in the form of a logistics network. A logistics network typically consists of a set of suppliers, a set of manufacturing centres, a set of warehouses, a set of distribution centres, and a set of retail outlets as well as channels for the flow of raw materials, work-in-process inventory, and finished products between the facilities [22]. Tavasszy [24] stated that logistics deals also with the achievement of customer satisfaction at the minimum level of costs. It is a crucial problem in business nowadays due to the high proportion of logistics costs in the costs of goods sold, often ranges from 10 to 35 per cent [5]. Geunes [27] also stated that logistics cost represent a large portion of total supply chain cost, especially when the supply chain network is extended to global market. Reducing the total cost while keeping the quality of network performance is a challenge in real business nowadays. Many researchers advocate a quantitative approach to improve and optimize logistics network performance [4,5, 7, 11, 14, 17].

The modern business paradigm has changed due to the global competitive market towards a network-based collaboration and integrated network. The integrated logistics network is the integration of several business functions (procurement, manufacturing, and distribution) that could be drawn as an abstract of nodes and arcs. It covers either micro (within facility) or macro levels (between facilities). This functions link the company with its customers and suppliers [5]. In industry practices, the integration display many benefits that include reduced costs, increased profit, increased market share, a strengthen competitive position, and the enhancement of the value of the company.

To remain competitive in the market then role of logistics optimization becomes more crucial than before. Businesses need to improve their network performance and reducing the costs in integrated analysis rather than individual ones. In integrated analysis, company's logistics functions are treated simultaneously rather than individual, and the network performance optimality will be achieved when each component is optimum as well as the global optimum, while the main objective is to satisfy the customers demand as well as maximize its profits. To achieve this, we must capture and integrate all logistics aspects in one integrated network model, than optimize a given performance measure and satisfy a given set of constraints [15]. Several papers [1, 8, 9, 10, 17, 22, 24, 28, 29] addressed this integrated network problem.

In this paper, we study logistics network that consists of a set plants, a set of warehouses, and a set of customer points. Each of customers has a given demand in each period in the planning horizon. We study both single-product and multi-product model. The plants and warehouses have known and finite capacity in certain period. All costs (setup cost, unit production cost, inventory cost, warehouse cost, and transportation cost) are deterministic.

This paper is organized as follows. Section 2 presents the literature review. Section 3 presents the model formulation in form of a mixed integer linear programming (MILP) for the single-product problem as well as the multi-product problem. Computational results are discussed in Section 4.

2. Literature Review

Several reviews on integrated models have been published [8, 12, 30, 31]. Erenguc et.al [12] provided a taxonomical framework for analyzing integrated production-distribution network in supply chains. Karimi et.al [30] provide a review of models and algorithms for the capacitated single-level lot sizing problem including variants and solution approaches in form of exact and heuristics algorithm. Brahimi et.al [31] provides a review problem of the single item lot sizing problem. They survey various solution methods and four different mathematical programming formulations of the classical problem, as well as the extensions for real-world application. Cohen and Lee [8] provide review of a model framework and an analytical procedure for evaluating the performance of production-distribution system. They addressed a methodology that measures tradeoff of cost, service, and flexibility in a production-distribution system. The methodology considers relationship between production and distribution control policies that affect inventory control, plant production mix, and production scheduling.

An integrated analysis is commonly used in logistics design network problems. Ambrosino and Scutella [1] addressed an integrated distribution network model that involves facility location, transportation, and inventory decision to minimize the associated cost by defining the number and the location of the facilities in the network. Two scenarios have been investigated, both of them do not include inventories, and two types customer are addressed: clients and big clients. Cordeau, et.al [9] formulated the deterministic logistics network in a single country and a single period that integrated location and capacity choices for plants and warehouses. In order to solve the logistics network design problem, two approaches are used, a simplex-based branch and bound and a Benders decomposition approach. Some valid constraints are also proposed to strengthen the LP relaxation of the problem. Introducing valid constraints in the master problem could dramatically reduce the number of cuts. Flipo and Finke [13] studied a multi-facility, multi-product and multi-period problem. They developed a network flow model with relatively few additional 0-1 variables to describe the linking constraints between periods. Park [22] presented the solution for integrated production and distribution planning and investigated the effectiveness of the integration through a computational study, in a multi-plant, multi-retailer, multi-item, and multi-period logistic environment where the objective is to maximize the total net profit. He developed mixed-integer models and a heuristics for solving the problem. The result of analysis shows that the heuristics perform well in term of both optimality approximation and computational time. It increases an average 4.1% in total net profit and 2.1% in demand fill rate. The analyses also show that the value of integrated planning was especially high in an environment of sufficiently large production capacity, high fixed cost, small vehicle capacity, and high unit stock out cost.

Analysis on the integration of facility location models with logistical functionality has also been used in solving logistics network problem. Syam [24] addressed an integrated model of logistics network that minimizes the total physical distribution costs by simultaneously determining optimal plants and warehouse locations, flows in the resulting network, shipment compositions and shipment frequencies in the network using simulated annealing to determine the optimal sets of open plants and warehouses, and Lagrangian relaxation to solve the flow and consolidation problem in the resulting network. Pirkul and Jayaraman [33] have investigated capacitated and bi-echelon network problem, and both include multiple plants, warehouses, and destinations. Ravi and Sinha [34] provide variants of multicommodity facility location and approximation algorithms.

Recently, several papers provide an analysis of the performance of integrated logistics network. Brahimi et.al [21] presented a capacitated multi-item lot-sizing problem which encounter time windows and capacity constraints as an extended model of [31]. The problem considered in their paper is the single-level single-resource multi-item capacitated lot-sizing problem with a finite planning horizon. Two mixed integer linear programming formulations are presented: the aggregate formulation and the facility location-based. They used lagrangian heuristics with two different experimental problems, noncustomer-specific problems and customer-specific problem. They found that the best heuristics are obtained when only capacity constraints are relaxed. Norden and Velde [32] studied multi-product lot-sizing problem with transportation capacity reservation contract. The problem is to determine transportation lot-sizes to meet warehouses demand with no backorder allowed and to minimize total cost, the sum of inventory carrying, ordering, and transportation costs. They proved that the problem is NP-hard. In this paper, they developed an integer linier programming formulation and a lagrangian algorithm for computing lower and upper bound on the optimal solution value.

3. Model Formulation

In order to improve logistics network performance, several optimization methods can be used, such as mathematical programming, genetic algorithms, simulated annealing, etc. This paper will use mathematical programming for the reason: it provides insight into problem, its characteristics and the linkages between the various interacting factors. Furthermore, there has been considerable progress in recent years with solving large-scale integer programming problems.

3.1. The Basic Model

We consider in this section a single product, multi period, multi facilities logistics network with the following features:

- l plants $P_1, P_2, \ldots, P_l$ where the product can be produced,
- m warehouses $W_1, W_2, \ldots, W_m$ where the product can be stored,
- n customer locations $C_1, C_2, \ldots, C_n$ where the product is required,
- Plant production capacities and warehouses storage capacities are known,
- Customer requirement at each centre are deterministic and known for each period. Furthermore, they must be met, that is backorders are not permitted,
- A planning horizon of T periods,
- A homogeneous fleet of vehicle transport the product through the network

Figure 3.1 depicts the situation. In this model, it is assumed that there are a number of plants that produce single product with a specific capacity over a period time. The set-up cost is a fixed cost on a lot-for lot basis, not dependent on the realized volume. It is incurred at each plant whenever the production runs. All products are assumed directly to be delivered to warehouse or retail outlet. Products are delivered using a homogeneous fleet vehicle. The movement of vehicle incurs a variable transportation cost only.

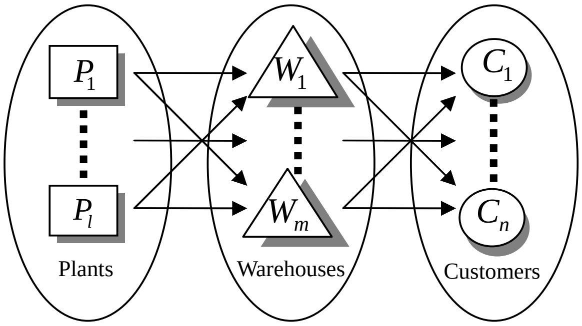

Fig 3.1. Multi-facilities network flow model

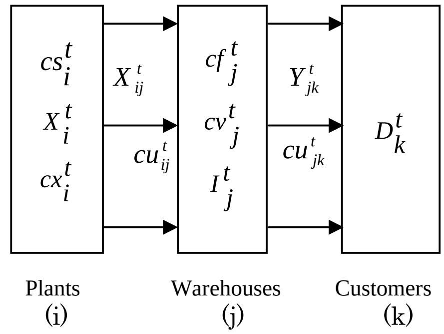

Fig 3.2. Network model representative

The demand for an item in a period at warehouse is expressed as a forecasted real demand. It is assumed that the demands are given and backordering is not allowed. Each warehouse must keep a limited amount of inventory, with higher holding cost.

The problem is to determine a production and distribution plan over the planning horizon that meets the customer demands, satisfies the capacity restrictions and minimizes the total logistics costs. The costs include: production, transportation, and inventory holding.

We represent the problem in the form of a network (Fig.3.2). We define the network for the flow of products from their production points to customers through the warehouse as storage. This model then refers to three components: the production sites, indexed by i, the warehouse, indexed by j, and the customers, indexed by k.

A mixed integer linear programming (MILP) model is then developed to solve the problem. The model comprises two cost components: (a) from Plants to Warehouses and (b) from Warehouses to Customers.

Parameters

Notation that we use in the model:

- P_i : capacity of plant i , i = 1,2, …, l
- W_j : capacity of warehouse j, j = 1,2, …, m
- D_k^t : demand of customer k in period t, k = 1,2, …, n; t = 1,2, …, T
- I_j^t : inventory level at warehouse j at the end of period t, j = 1,2, …, m; t = 1,2, …, T
- cs_i^t : setup cost at plant i in period t, i = 1,2, …, l ; t = 1,2, …, T
- cx_i^t : unit cost of production at plant i in period t, i = 1,2, …, l; t = 1,2, …, T
- ct_{ij}^t : unit cost of transportation to delivered product from plant i to warehouse j in period t, i = 1,2, …, l; j = 1,2, …, m; t = 1,2, …, T
- cu_{jk}^t : unit cost of transportation to delivered product from warehouse j to customer k in period t, j = 1,2, …, m; k= 1,2, …, n; t = 1,2, …, T
- cf_j^t : unit cost of space used at warehouse j in period t, j = 1,2, …, m; t = 1,2, …, T
- cv_j^t : variable cost at warehouse j in period t, j = 1,2, …, m; t = 1,2, …, T
- su_j^t : amount space used at warehouse j in period t, j = 1,2, …, m; t = 1,2, …, T
- s_j^t : amount space leased/available at warehouse j in period t, j = 1,2, …, m; t = 1,2, …, T
- M : a big M number

Decision Variables

- X_i^t : amount of product produced at plant i in period t
- X_{ij}^t : amount of product transported from plant i to warehouse j in period t
- Y_{jk}^t : amount of product transported from warehouse j to customer k in period t
- I_j^t : inventory level at warehouse j at the end of period t

Objective Function

The objective is to minimize the total cost of production, transportation, and inventory over the T periods. The model assumes no starting inventory. The model can be expressed as follows:

Minimize

$$\sum_{t=1}^{T}\sum_{i=1}^{l} z_i^t cs_i^t + cx_i^t X_i^t + \sum_{t=1}^{T}\sum_{i=1}^{l}\sum_{j=1}^{m} X_{ij}^t ct_{ij}^t + \sum_{t=1}^{T}\sum_{j=1}^{m} cf_j^t s_j^t + \sum_{t=1}^{T}\sum_{j=1}^{m} I_j^t cv_j^t + \sum_{t=1}^{T}\sum_{j=1}^{m}\sum_{k=1}^{n} Y_{jk}^t cu_{jk}^t \tag{3.1}$$

where,

$$z_i^t = \begin{cases} 1, & \text{if } X_i^t > 0, \\ 0, & \text{else} \end{cases}$$

Model Constraints

$$X_i^t \le M\, z_i^t \qquad \forall i,t \tag{3.2}$$

$$\sum_j X_{ij}^t \le P_i^t \qquad \forall i,j,t \tag{3.3}$$

$$\sum_j X_{ij}^t \le X_i^t \qquad \forall i,j,t \tag{3.4}$$

$$D_k^t = \sum_j Y_{jk}^t \qquad \forall j,k,t \tag{3.5}$$

$$\sum_i X_{ij}^t + I_j^{t-1} - \sum_k Y_{jk}^t \le W_j^t \qquad \forall i,j,k,t \tag{3.6}$$

$$\sum_i X_{ij}^t + I_j^{t-1} \ge \sum_k Y_{jk}^t \qquad \forall i,j,k,t \tag{3.7}$$

$$\sum_i X_{ij}^t + I_j^{t-1} = \sum_k D_k^t + I_j^t \qquad \forall i,j,k,t \tag{3.8}$$

$$su_j^t = \sum_{ji} X_{ij}^t - \sum_{jk} Y_{jk}^t + I_j^{t-1} \qquad \forall i,j,k,t \tag{3.9}$$

$$su_j^t \le s_j^t \qquad \forall j,t \tag{3.10}$$

$$X_i^t, \quad X_{ij}^t, \quad Y_{jk}^t, \quad I_j^t \quad \ge 0 \tag{3.11}$$

$$P_i^t, \quad W_j^t, \quad \ge 0 \tag{3.12}$$

$$z_i^t = 0,1 \tag{3.13}$$

The objective function (3.1) represents the total costs over the T periods. Constraint (3.2) assures that a setup cost will be incurred if there is product produced in plant *i*. It will guarantee that z_i^t takes value one whenever X_i^t is positive. Number of product produced at plant *i* in period t and transferred from plant *i* to warehouses *j* in period t is restricted by the capacity constraints (3.3) and (3.4). Constraint (3.5) requires that warehouses must satisfy all demand. Product flow from plant *i* to warehouse *j* must respected the throughput capacity of warehouse *j*, as indicated in (3.6). Constraint (3.7) related to product flow requirement in warehouse *j*. Total product delivered out from warehouse cannot exceed total product delivered to warehouse. Balance constraints of inventory level in warehouse *j* and warehouse space used during period time t are provided by (3.8), (3.9), (3.10). Constraints (3.11) and (3.12) are non-negative value. Constraint (3.13) is zero-one variable.

Multi periods issue

Analysis of multi-period issue is mostly used to anticipate periods of high demand. Therefore, it is necessary to store reasonable amount of product in advance, which in our model, this stock will be in warehouses. In this case, we need arcs between nodes representing a same stock during adjacent periods. The product flows on these arcs are products that stay in stock from one period to the next. As for that, there should be holding costs associated with these flows.

Constraints (3.6) and (3.8) are used to control inventory flow in warehouses as well as its capacity.

3.2. Extended Model Problem

In this section, we extend the problem formulation described in Section 3.1 by introducing the capacity of transportation facilities, and by allowing backorders.

We now introduce the capacity of transport facilities. The transport capacity in this model is the limitation of the maximum quantity of product that can be carried out by the vehicle in period T. It is assumed that:

- The limitation is due to physical constraints and availability of transport facilities,
- The capacity of each transportation lines could be identical or different.

Let T_{ij}^t represent the capacity of transportation line between plant *i* and warehouse *j* in period t, and U_{jk}^t represent the capacity of transportation line between warehouse *j* and customer *k* in period t. Then,

$$X_{ij}^t \le T_{ij}^t \qquad i = 1,2, \ldots, l;\ j = 1,2, \ldots, m;\ t = 1,2, \ldots, T \tag{3.14}$$

$$Y_{jk}^t \le U_{jk}^t \qquad i = 1,2, \ldots, l;\ j = 1,2, \ldots, m;\ t = 1,2, \ldots, T. \tag{3.15}$$

where

$$T_{ij}^t,\quad U_{jk}^t \quad \ge 0 \tag{3.16}$$

The model then can be expressed as follows:

Minimize (3.1)

Subject to (3.2) – (3.16)

Our rest extension is to allow backorders. In this case, it is assumed that when demand cannot be fully satisfied then the shortage is backlogged, and satisfied during the next period. Johnson [26] stated that to represent the possibility of satisfying demand in a period through production in a later period, we could use network flow model. If Ih_j^t is the on hand inventory at the end of period t at warehouse *j*, and Ib_j^t is backorder position at warehouse *j* in period t, then the net inventory level at warehouse *j* in period t is:

$$I_j^t = Ih_j^t - Ib_j^t \tag{3.17}$$

where

$$Ih_j^t, Ib_j^t \quad \ge 0 \tag{3.18}$$

Notation that used in the model:

Ib_j^t : backorder level at warehouse *j* in period t

Ih_j^t : on hand inventory at warehouse j in the end of period t

cb_j^t : backorder cost at warehouse j in period t

Then, our new model can be expressed as follows:

Minimize

$$\sum_{t=1}^{T}\sum_{i=1}^{l} z_i^t cs_i^t + cx_i^t X_i^t + \sum_{t=1}^{T}\sum_{i=1}^{l}\sum_{j=1}^{m} X_{ij}^t ct_{ij}^t + \sum_{t=1}^{T}\sum_{j=1}^{m} cf_j^t s_j^t + \sum_{t=1}^{T}\sum_{j=1}^{m} Ih_j^t cv_j^t + \sum_{t=1}^{T}\sum_{j=1}^{m} Ib_j^t cb_j^t + \sum_{t=1}^{T}\sum_{j=1}^{m}\sum_{k=1}^{n} Y_{jk}^t cu_{jk}^t \tag{3.19}$$

Subject to: (3.2) – (3.18)

3.3. Multi-product Model

In this section we extend our model to multi-products. Suppose there were *q* product identified as 1,2,…q, which can be produced in plants P over a period of time. The plant's production capacities for each product in a period are known. All products are assumed delivered directly to warehouse or retail outlet. Products are delivered using a homogeneous fleet vehicle with specific capacity for each product. The demand for product type *q* at warehouse *j* in a period *t* is expressed as a forecasted real demand. It is assumed that the demands are given and backordering is not allowed. Each warehouse could keep a limited amount of inventory for each product, with specific holding cost.

We formulate the problem as a multi product network flow problem with fixed charge cost function. The problem is to determine a production and distribution plan over the planning horizon that meets the customer demands, satisfies the capacity restrictions and minimizes the total logistics costs. The costs include: production, transportation, and inventory holding. It is assumed those initial inventories are zero for all items.

A mixed integer linear programming (MILP) model is then developed to solve the problem. The model assumes no starting inventory and not allowing backorders. The model can be expressed as follows:

Minimize

$$\sum_{t=1}^{T}\sum_{q=1}^{q}\sum_{i=1}^{l} z_{iq}^{t} cs_{iq}^{t} + X_{iq}^{t} cx_{iq}^{t} + \sum_{t=1}^{T}\sum_{q=1}^{q}\sum_{i=1}^{l}\sum_{j=1}^{m} X_{ijq}^{t} ct_{ijq}^{t} +$$
$$\sum_{t=1}^{T}\sum_{q=1}^{q}\sum_{j=1}^{m} s_{jq}^{t} cf_{jq}^{t} + \sum_{t=1}^{T}\sum_{q=1}^{q}\sum_{j=1}^{m} I_{jq}^{t} cv_{jq}^{t} + \sum_{t=1}^{T}\sum_{q=1}^{q}\sum_{j=1}^{m}\sum_{k=1}^{n} Y_{jkq}^{t} cu_{jkq}^{t} \tag{3.20}$$

Subject to:

$$X_{iq}^{t} \le M \cdot z_{iq}^{t} \qquad \forall i,q,t \tag{3.21}$$

$$\sum_{iq} X_{ijq}^{t} \le P_{iq}^{t} \qquad \forall i,j,q,t \tag{3.22}$$

$$\sum_{jq} X_{ijq}^{t} \le X_{iq}^{t} \qquad \forall i,j,q,t \tag{3.23}$$

$$\sum_{q} X_{ijq}^{t} \le T_{ij}^{t} \qquad \forall i,j,q,t \tag{3.24}$$

$$\sum_{q} Y_{jkq}^{t} \le U_{jk}^{t} \qquad \forall j,k,q,t \tag{3.25}$$

$$D_{kq}^{t} = \sum_{jq} Y_{jkq}^{t} \qquad \forall i,j,q,t \tag{3.26}$$

$$D_{k}^{t} = \sum_{q} D_{kq}^{t} \qquad \forall j,k,q,t \tag{3.27}$$

$$\sum_{iq} X_{ijq}^{t} + I_{jq}^{t-1} - \sum_{kq} Y_{jkq}^{t} \le \cdot W_{jq}^{t} \qquad \forall i,j,k,q,t \tag{3.28}$$

$$\sum_{iq} X_{ijq}^{t} + I_{jq}^{t-1} \ge \sum_{kq} Y_{jkq}^{t} \qquad \forall i,j,k,q,t \tag{3.29}$$

$$\sum_{iq} X_{ijq}^{t} + I_{jq}^{t-1} = \sum_{kq} Y_{jkq}^{t} + I_{jq}^{t} \qquad \forall i,j,k,q,t \tag{3.30}$$

$$su_{jq}^{t} = \sum_{jiq} X_{ijq}^{t} - \sum_{jkq} Y_{jkq}^{t} + I_{jq}^{t-1} \qquad \forall i,j,k,q,t \tag{3.31}$$

$$su_{jq}^{t} \le s_{j}^{t} \qquad \forall j,q,t \tag{3.32}$$

$$X_{iq}^{t}, X_{ijq}^{t}, Y_{jkq}^{t}, I_{jq}^{t}, T_{ij}^{t}, U_{jk}^{t} \ge 0 \tag{3.33}$$

$$P_i^t\,,\quad W_j^t\,,\quad \geq 0 \tag{3.34}$$

$$z_{iq}^t \;= 0,1 \tag{3.35}$$

where,

$$z_{iq}^t = \begin{cases} 1 & \text{if } X_{iq}^t > 0, \\ 0 & \text{else} \end{cases}$$

The objective function (3.20) represents the total costs over the T periods. Constraint (3.21) assures that a setup cost will be incurred if there is product type *q* produced in plant *i*. Number of product produced at plant *i* in period t and transferred from plant *i* to warehouses *j* in period t is restricted by the capacity constraints (3.22) and (3.23). Product transferred from plant *i* to warehouse *j* and from warehouse j to customer *k* are restricted by the transportation capacity constraints (3.24 and 3.25). Constraint (3.26) and (3.27) requires that warehouses must satisfy all demand. Product flow from plant *i* to warehouse *j* must respected the throughput capacity of warehouse *j*, as indicated in (3.28). Constraint (3.29) related to product flow requirement in warehouse *j*. Total product delivered out from warehouse cannot exceed total product delivered to warehouse. Balance constraints of inventory level in warehouse *j* and warehouse space used during period time t are provided by (3.30), (3.31), and (3.32). Constraint (3.33) and (3.34) is the non-negative value. Constraint (3.35) is zero-one variable.

4. Computational Results

In this section, we describe the computational experience in solving the single product and multi product models. We generate several test data to demonstrate the applicability of the mathematical formulations above. The following sample data are used:

- There are 7 nodes which represent 2 Plants (Plant1 and Plant2), 3 Warehouses (Warehouse1, Warehouse2, and Warehouse3), and 2 Customers (Customer1 and Customer2),
- T period observation is 9 period,
- Plants production capacity are in the range 2000 – 6000 unit/period,
- Inventory storage capacity in warehouses are in the range 2000 – 5000 unit/period,
- Demand model are as shown in Table 4.1,
- Setup cost model and production cost model in production facilities are as shown in Table 4.2,
- Cost incurred in warehouses are shown in Table 4.3,
- Transportation cost between plants, warehouses, and customers are indicated in Table 4.4.

Table 4.1. Customer demand (in Unit)

Period	Demand Model 1		Demand Model 2	
	Customer 1	Customer 2	Customer 1	Customer 2
1	1800	0	1800	2000
2	2000	500	2000	2500
3	1500	1000	1500	2000
4	2700	2000	2700	2000
5	2000	1800	2000	2800
6	800	1600	1800	2600
7	1300	1900	1300	2900
8	2300	1500	2300	2500
9	2500	2100	2500	2100

Table 4.2. Plants Cost data (in $)

Period	Cost Model 1				Cost Model 1			
	Plant1		Plant1		Plant2		Plant2	
	(a)	(a)	(b)	(a)	(b)	(b)	(a)	(b)
1	8000	50	6000	40	180000	50	160000	40
2	8000	50	6000	40	180000	50	160000	40
3	8000	50	6000	40	180000	50	160000	40
4	8500	55	6000	40	185000	55	160000	40
5	8500	55	7500	40	185000	55	175000	40
6	8500	55	7500	40	185000	55	175000	40
7	8500	60	7500	40	185000	60	175000	40
8	8500	60	7500	55	185000	60	175000	55
9	8500	60	7500	55	185000	60	175000	55

(a): Setup Cost ($); (b): Production Cost ($)/unit.

Table 4.3. Warehouses Cost data (in $)

Period	Warehouse1		Warehouse2		Warehouse3	
	Space Cost ($)/M3	Variable Cost ($)/unit	Space Cost ($)/M3	Variable Cost ($)/unit	Space Cost ($)/M3	Variable Cost ($)/unit
1	20	10	18	12	20	8
2	20	10	18	12	20	8
3	20	10	18	12	20	8
4	20	10	18	12	20	8
5	20	10	18	12	20	8
6	20	10	18	12	20	8
7	20	10	18	12	20	8
8	22	12	19	14	22	10
9	22	12	19	14	22	10

Table 4.4. Transportation cost data (in $)

$/ unit	Plant1	Plant2	Customer1	Customer2
Warehouse1	30	50	40	50
Warehouse2	90	10	80	60
Warehouse3	70	40	70	20

By varying any combination of capacity constraints, demand models, and production cost, inventory cost as well as transportation cost, we generated 36 test problems. To illustrate the impact of different factors on network with capacitated facilities, we consider 4 different configurations of demands and setup cost. Then, for each configuration we have 3 scenarios with production capacity changes and 4 scenarios in term of warehouse capacity. In all configurations, we assume that there are no changes in other cost parameters as shown in Table 4.3 and 4.4.

The optimal product produced and product movements as well as the optimal total costs are computed for each configuration under all scenarios. The experiments were conducted on personal computer with Pentium IV 2 GHz and 512 Mbyte memory.

In all configuration scenarios, we observe that the optimal solution is obtain when the capacity constraints not to small. If we put too small capacity value, it will force model not to find the optimal solution. We observe a reduce in total costs when we change the capacity of plants and warehouses.

We find a change in system behavior when we increase the setup cost very high. The system tends to increase the production level to maximum capacity and increase total inventory hold in several periods. The maximum values of total cost reduction are achieved when we increase the warehouse capacity to 4000 units. No cost reduction happens beyond that capacity value, see (a) in Fig 4.1. However, if we increase the setup cost very high and we increase plant capacity, we will obtain total cost reduction in all warehouse capacity observed ((b) in Fig 4.1.). Solution time will also increase as we increase the set of capacity, however the value still below 0.01 cpu seconds.

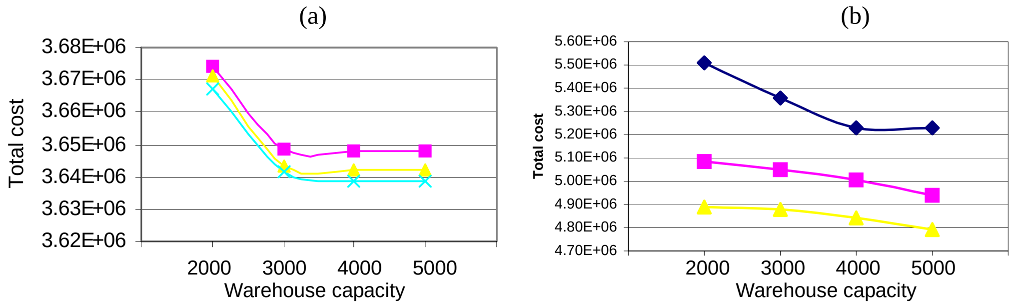

Fig. 4.1. Effect of setup cost with total cost

When we extend our model by adding the transportation network capacity on base model, we observe that the total cost value increases. For example, with plant1 and plant2's capacity: 4000 unit and 3000 unit respectively, and warehouse capacity: 3000 unit, total cost value increase from 3.65M to 3.98M if we limit the maximum product that can be transported in 1000 unit.

To observe the impact of capacitated transportation line on the increment of total cost, we experiment with different arch capacities, i.e. 25%, 50%, 75% of the warehouses capacity. From computational we found that, as the capacity of transportation line getting tight, the total cost will increase higher. The behavior of inventory holding in warehouses is correlated with the value of transportation capacity. As we reduce the transport capacity, warehouses tend to create more inventories over period T. The system create inventory in warehouses almost in all periods.

Table 4.5. Data test with 50% transportation capacity and without

Warehouse Cap	**Total Cost With 50% Warehouse Capacity (a)**	**Total Cost with Capacity= 1500 (b)**	**Total Cost with No Capacity (c)**	**% [(a)-(c)]/(c)**	**% [(b)-(c)]/(c)**
2000	4.00E+06	3.7472E+06	3.67E+06	9.01%	1.99%
3000	3.73E+06	3.7323E+06	3.65E+06	2.30%	2.30%
4000	3.67E+06	3.7323E+06	3.65E+06	0.71%	2.31%
5000	3.65E+06	3.7323E+06	3.65E+06	0.08%	2.31%

Table 4.5 displays the effect of transportation capacity on total cost with respect to warehouse capacity. Value in (a) is the total cost if we use 50% warehouse capacity as transportation capacity, value in (b) is the total cost if we use fixed transportation capacity = 1500 unit, and value in (c) is the total cost with no transportation capacity. From this result, if we use fixed transportation capacity (1500 unit), the value of total cost increase 2.22% (average) as the warehouse capacity increase. It will increase as the warehouse capacity increase.

When we allow the system to handle backorders, our computational results show that the problem getting harder to solve. We experiment with any combination of warehouses capacity and demand model. The warehouses capacity is between 2000-5000 with 3 demand models. We found that as the warehouses capacity increase the total cost will decrease. Using the small capacity of warehouses also will produce high error gap (average 2.07%) as compare to other (average 0.42%). As our data still small, the maximum running is around 2.00 cpu seconds.

To illustrate the impact of different factors on network with capacitated facilities and multi product, we consider 2 different configurations of demands and setup cost. Then, for each configuration we have 3 scenarios with production capacity changes and 3 scenarios of warehouse capacity changes. In all configurations, we assume that there are no changes in other cost parameters.

Similar with our result in single-product model, in all scenarios of our multi-product test data we observe that the optimal solution is obtain when the capacity constraints not to small. Using too small capacity value, it will force model not to find the optimal solution. In our cases, when we set the plants capacity below 4000 units, no integer solution found. Therefore, to analyze our system behavior, we use capacity higher than 4000 units.

As we increase the production capacity as well as warehouse, total cost will reduce. If we look at the effect of setup cost changes, when we increase the setup cost very high again the system tends to increase the production level to maximum capacity and increase total inventory hold in several periods. This situation is similar with single item model.

5. Conclusion

In this paper, models to deal with single-product multi-facilities multi-periods as well as multi products that used to improve logistics network performance have been presented. Our logistics network model is an integrated model in a two-stage that consists of a set plant, a set of warehouse, and a set of customer point. The plants produce product to satisfy customer demand over period T where demand is expressed as a forecasted real demand. We optimize the network performance by minimizing its total cost (production, transportation, and inventory holding costs), and plan the inventory at warehouses over period of time. We model the problem as a network flow problem.

We found from the computational result, base model can obtain the optimal solution better if the capacities of facilities are increased. Using too small capacity value will force model not to find the optimal solution. Any combination of facilities capacity increment can reduce the total cost. In the following, by introducing transportation capacity and backordering, it will increase the objective value and solution time as the problem getting harder to solve. We find that the behavior of inventory holding in warehouses is correlated with the value of transportation capacity. As we reduce the transport capacity, warehouses tend to create more inventories over period T. Similar with our result in single-product model, in all scenarios of our multi-product test data we observe that the optimal solution is obtain when the capacity constraints not to small. As we increase the production capacity as well as warehouse, total cost will reduce.

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