Lanjutan Tugas Kuliah 2

Disini Anda harus mengupload PPT.

PENYELESAIA TUGAS

1/ Tentukan Deret Taylor dan Macluoren dari :

a. $F(x) = \frac{1}{1+x}$

$$F^{(0)}(x) = \frac{1}{1+x}$$

$$F^{(1)}(x) = \frac{-1}{(1+x)^2}$$

$$F^{(2)}(x) = \frac{2}{(1+x)^3}$$

$$F^{(3)}(x) = \frac{-6}{(1-x)^4}$$

$\ldots\ldots$

Untuk itu Ditemukan Pola.

$$F^{(n)}(x) = \frac{(-1)^n n!}{(1+x)^{n+1}}$$

Dan $F^{(n)}(0) = \frac{(-1)^n n!}{(1+0)^{n+1}} = (-1)^n n!$

Defenisi Deret Macluoren

$$F(x) = \sum_{n=0}^{\infty} \frac{F^{(n)}(0)}{n!} x^n$$

$$F(x) = \sum_{n=0}^{\infty} \frac{(-1)^n n!}{n!} x^n = \sum_{n=0}^{\infty} (-1)^n x^n$$

$$F(x) = \sum_{n=0}^{\infty} (-1)^n x^n$$

$$F(x) = 1 - x + x^2 - x^3 + \ldots$$

b. $F(x) = \ln(1+x)$

$F(0) = \ln 1 = 0.$

$F'(x) = (1+x)^{-1} \Rightarrow F'(0) = 1.$

$F''(x) = -(1+x)^{-2} \Rightarrow F''(0) = -1 = 1!$

$F'''(x) = 2(1+x)^{-3} \Rightarrow F'''(0) = 2 = 2!$

$F^{IV}(x) = -6(1+x)^{-4} \Rightarrow F^{IV}(0) = -3 = -3!$

$F^{V}(x) = 24(1+x)^{-5} \Rightarrow F^{V}(0) = 24 = 4!$

Maka $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \ldots$

Di rangkum.

1. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \quad -\infty < x < \infty.$
2. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \ldots -\infty < x < \infty.$
3. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \ldots \quad -\infty < x < \infty.$
4. $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots \quad -1 < x \leq 1,$

Dengan cara yang sama akan di peroleh Deret.

5. $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} \ldots \quad |x| < \pi/2.$
6. $\cot x = 1/x - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} \ldots \quad 0 < |x| < \pi.$
7. $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} \ldots \quad -\infty < x < \infty.$
8. $\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} \ldots \quad -\infty < x < \infty.$

2) Tentukan Deret Taylor dan Maclaurin dari :

a. $F(x) = \sin x$

$F(0) = 0.$

$F'(x) = \cos x \Rightarrow F'(0) = 1.$

$F''(x) = \sin x \Rightarrow F''(0) = 0.$

$F'''(x) = \cos x \Rightarrow F'''(0) = -1$

$F^{iv}(x) = \sin x \Rightarrow F^{iv}(0) = 0$

$F^{v}(x) = \cos x \Rightarrow F^{v}(0) = 1$

Maka $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots$

b. $F(x) = \cos x$.

$F(0) = 1$

$F'(x) = -\sin x \Rightarrow F'(0) = 0$

$F''(x) = -\cos x \Rightarrow F''(0) = -1$

$F'''(x) = \sin x \Rightarrow F'''(0) = 0$

$F^{iv}(x) = \cos x \Rightarrow F^{iv}(0) = 1$

$F^{v}(x) = \sin x \Rightarrow F^{v}(0) = 0$

Maka $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$

Jayapura

Nim. 2027110010

No.
Date : 5-10-2020

NAMA : SELAMAT, ST
NIM : 202710025
KELAS : MT-S5A1

SOAL

1. TENTUKAN DERET TAYLOR & MACLUOREN DARI

a. $f(x) = \frac{1}{1+x}$ b. $f(x) = \ln(1+x)$

2. TENTUKAN DERET TAYLOR & MACLUOREN DARI

a. $f(x) = \sin x$ b. $f(x) = \cos x$

JAWABAN

1. a. $f(x) = \frac{1}{1+x}$

DERET MACLOUREN BERARTI PUSATNYA $x_0 = 0$

SEBELUMNYA KITA SUDAH MEMPUNYAI DERET KHUSUS 1 YAITU:

$$f(x) = \frac{1}{1-x} = \sum_{n=0}^{n} x^n$$

DARI BENTUK DIATAS INILAH KITA MENENTUKAN DERET MACLUOREN

$$\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{n} (-x)^n = \sum_{n=0}^{n} \{(-).(x)\}^n = \sum_{n=0}^{n} (-1)^n x^n$$

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b. $f(x) = \ln(1+x)$

TURUNAN DARI $f(x) = e^x$, $f'(x) e^x$, $f''(x) = e^x$ .... DST
DERET MACLAUREN UNTUK e^x ADALAH

$$e^x = e^{(0)} + \frac{(x-0)}{1!} e^0 + \frac{(x-0)^2}{2!} e^{(0)} + \frac{(x-04)^3}{3!} e^{(0)} + \frac{(x-0)^4}{4!} e^{(0)} + \dots$$

$$= 1 + x + \frac{x2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} \dots$$

TURUNAN FUNGSI $\ln(1+x)$ YAITU :

$$f(x) = \ln(x+1) \Rightarrow f'(x) = (x+1)^{-1} \Rightarrow f'(x) = -(x+)^{-2}$$
$$\Rightarrow f'''(x) = 2(x-1)^{-3} \Rightarrow f^{(4)}(x) = -6(x+1)^{-4} \dots \text{dst}$$

DERET MACLAUREN DARI $\ln(x+1)$ ADALAH ;

$$\ln(x+1) = \ln(0+1) + \frac{(x-0)}{1!}(0+1)^{-1} + \frac{(x-0)^2}{2!}\left(-(0+1)\right)^{-2} + \frac{(x-0)^3}{3!} 2(0+1)^{-3}$$
$$+ \frac{(x-0)^4}{4!}\left(-6(0+1)^{-4}\right) + \dots$$
$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

2. a. $f(x) = \sin(x)$

FUNGSI $f(x) = \sin(x)$ KEDALAM DERET TAYLOR DISEKITAR $x_0 = 1$
TENTUKAN TURUNAN $\sin(x)$ YAITU :

$$f(x) = \sin(x) \Rightarrow f'(x) = \cos(x), \Rightarrow f''(x) = -\sin(x)$$
$$\Rightarrow f^3(x) = -\cos(x) \Rightarrow f^4(x) = \sin(x) \dots \text{DST}$$

SEHINGGA $\sin(x)$ DIHAMPIRI DENGAN DERET TAYLOR ADALAH

$$\sin(x) = \sin(1) + \frac{(x-)}{1!}\cos(1) + \frac{(x-1)^2}{2!}(-\sin(1)) + \frac{(x-1)^3}{3!}(-\cos(1) + \dots$$
$$+ \frac{(x-1)^4}{4!}\sin(1) + \dots$$

MISALKAN $x - 1 = h$, MAKA :

$$\sin(x) = \sin(1) + h\cos(1) - \frac{h^2}{2}\sin(1) - \frac{h^3}{6}\cos(1) + \frac{h^4}{24}\sin(1) + \dots$$
$$= 0{,}84 + 0{,}54\,h - 0{,}420\,h^2 - 0{,}09\,h^3 + 0{,}03\,h^4 + \dots$$

No.

Date :

b. $f(x) = \cos x$

$$\Rightarrow f(x) = \cos x \rightarrow f(0) = 1$$
$$f'(x) = -\sin x \rightarrow f'(0) = 0$$
$$f^2(x) = -\cos x \rightarrow f^2(0) = -1$$
$$f^3(x) = \sin x \rightarrow f^3(0) = 0$$
$$f^4(x) = \cos x \rightarrow f^4(0) = 1$$ DAN SETERUSNYA

JADI $f(x) = \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \ldots\ldots$

HARUS DITUNJUKKAN BAHWA $\lim\limits_{n \to \infty} : S_n = 0$

$$S_n = \frac{x^n}{n:} - f^{(n)}(tx) = \frac{x^n}{n!} - \cos\left(tx + \frac{1}{2} n\pi\right)$$

JADI $|S_n| = \frac{|x^{tx}|}{n:} \left|\cos\left(tx + \frac{1}{2} n\pi\right)\right|$ KARENA $\left|\cos\left(tx + \frac{1}{2} n\pi\right)\right| \leq 1$

MAKA $\lim\limits_{n \to \infty} S_n = 0$ UNTUK SETIAP NILAI X.

PAPERLINE

1. Tentukan deret Taylor dan Macluoren dari

a. $f(x) = \frac{1}{1+x}$

Jawaban

$f^{(0)}(x) = \frac{1}{1+x}$

$f^{(1)}(x) = \frac{-1}{(1+x)^2}$

$f^{(2)}(x) = \frac{2}{(1+x)^3}$

$f^{(3)}(x) = \frac{-6}{(1+x)^4}$

. . . . . .

Dari itu ditemukan pola

$f^{(n)}(x) = \frac{(-1)^n n!}{(1+x)^{n+1}}$

Dan

$f^{(n)}(x) = \frac{(-1)^n n!}{(1+0)^{n+1}} = (-1)^n n!$

Dengan Definisi Deret Macluoren

$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$

$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n n!}{n!} x^n = \sum_{n=0}^{\infty} (-1)^n x^n$

$f(x) = 1 - x + x^2 - x^3 + \ldots$

b. $f(x) = \ln(1+x)$

Jawab

$f(0) = \ln 1 = 0$

$f'(x) = (1+x)^{-1} \longrightarrow f'(0) = 1$

$f''(x) = -(1+x)^{-2} \longrightarrow f''(0) = -1 = 1!$

$f'''(x) = 2(1+x)^{-3} \longrightarrow f'''(0) = 2 = 2!$

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Maka $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \cdots$

Dirangkuman

1. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \quad -\infty < x < \infty$
2. $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} \cdots \quad -\infty < x < \infty$
3. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \cdots \quad -\infty < x < \infty$
4. $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} \cdots \quad -1 < x < 1$

Dengan cara yang sama akan diperoleh deret

5. $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} \cdots \quad |x| < \pi/2$
6. $\cot x = \frac{1}{x} - \frac{x}{3} - \frac{x^3}{45} - \frac{2x^5}{945} \cdots \quad 0 < |x| < \pi$
7. $\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} \cdots \quad -\infty < x < \infty$
8. $\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \frac{x^8}{8!} \cdots \quad -\infty < x < \infty$

2. Tentukan Deret Taylor dan Macluoren

a. $f(x) = \sin x$

Jawab

$f(0) = 0$

$f'(x) = \cos x \longrightarrow f'(0) = 1$

$f''(x) = -\sin x \longrightarrow f''(0) = 0$

$f'''(x) = -\cos x \longrightarrow f'''(0) = -1$

$f^{iv}(x) = \sin x \longrightarrow f^{iv}(0) = 0$

$f^{v}(x) = \cos x \longrightarrow f^{v}(0) = 1$

Maka $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots$

b. $f(x) = \cos x$

$f(0) = 1$

$f'(x) = -\sin x \longrightarrow f'(0) = 0$

$f''(x) = -\cos x \longrightarrow f''(0) = -1$

$f'''(x) = \sin x \longrightarrow f'''(0) = 0$

$f^{iv}(x) = \cos x \longrightarrow f^{iv}(0) = 1$

$f^{v}(x) = -\sin x \longrightarrow f^{v}(0) = 0$

Maka $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots$

Nama : sujono

Nim : 20271002 0

2. a. Deret Taylor $\sin(x)$ adalah $(x_0 = 1)$

$$\sin x = \sin(1) + \frac{(x-1)}{2!}\cos(1) + \frac{(x-1)^2}{2!}(-\sin 1)$$

$$+ \frac{(x-1)^3}{3!}(-\cos(1)) + \frac{(x-1)^4}{4!}\sin(1) + \cdots$$

misalkan $x - 1 = h$ maka

$$\sin x = \sin(1) + h\cos(1) - \frac{h^2}{2}\sin(1) - \frac{h^3}{6}\cos(1)$$

$$+ \frac{h^4}{24}\sin(1) + \cdots$$

b. Deret Taylor $\cos(x)$

$$\cos(x) = \cos 1 + \frac{(x-1)}{1!}(-\sin 1) + \frac{(x-1)^2}{2!}(-\cos 1)$$

$$+ \frac{(x-1)^3}{3!}\sin 1 + \frac{(x-1)^4}{4!}\cos 1$$

misalkan $x - 1 = h$

$$\cos(x) = \cos 1 - h\sin 1 - \frac{h^2}{2}\cos 1 + \frac{h^3}{6}\sin 1$$

$$+ \frac{h^4}{24}\cos 1 + \cdots$$

Nama : Agus Putut
NIM : 202710030

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① Carilah Turunan pertama dari fungsi:

a. $f(x) = (2x^2 - 3)(2x^2 - 5x + 7)$

Turunan pertama nya bisa di cari dengan melakukan perhitungan seperti berikut.

$U = (2x^2 - 3) : U' = 2x$

$U' = (2x^2 - 5x + 7) : 0 = 2x - 5$

$f'(x) = U'v + v'U$

$= 2x(2x^2 - 5x + 7) + (2x - 5)(2x^2 - 3)$

$= 4x^3 - 10x\ 14x + 4x^2 - 6 - 10x^2 + 15$

$= 4x^3 + 4x - 6x^2 + 9$

b. $f(x) = \dfrac{2x - 1}{3 + x^2}$

Contoh $\cdot$ $U = 2x - 1 \quad : U' = 2$

$v = 3 + x^2 \quad : v = x$

$f(x) = \dfrac{U}{v}$ maka

$f'(x) = \dfrac{U'v - Uv'}{v^2}$

$= \dfrac{2(3 + x^2) - (2x - 1)x}{(3 + x^2)}$

$= \dfrac{6 + 2x^2 - 2x^2 - x}{(3 + x^2)^2}$

$= \dfrac{6x}{(3 + x^2)^2}$

Nama : Agus Putut
NIM : 202710030

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c $f(x) = 3(2x-4)^2$

. misal

$f(u) = 3u^2$ maka $f'(u) = 3u$

$u(x) = 2x-4$ $u'(x) = 2$

$f(x) = f(u \cdot u'(x)$

$= 3u \cdot 2$

$= 3(2x-4) \cdot 2$

$= 6(2x-4)$

$= 12x - 24)$

d $f(x) = 5 \cos x - \frac{1}{2} x 2$

$f(0) = 5\cos(0) - \frac{1}{2} \times 0^2$

$f(0) = 5 \times 1 - \frac{1}{2} \times 0^2$

$f(0) = 5\cos(0) - \frac{1}{2} \times 0^2$

0 yang di pangkatkan bilangan positif sama dengan 0

$f(0) = 5 \times 1 \frac{1}{2} \times 0$

$f(0) = 5 - \frac{1}{2} \times 0$

$f(0) = 5 - 0$

$f(0) = 5$

Nama : Agus Putut
NIM : 202710030

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Carilah integral fungsi berikut

a. $\int \frac{2x-1}{x^2} dx$

$= \int \frac{2x-1}{x^2} dx = \int 2x - \int \frac{1}{x^2}$

$= 2\int \frac{1}{x} dx - \int \frac{1}{x^2} dx$

$= 2\int \frac{1}{x} dx - (\frac{1}{-1})x^{-1} + c$

$= 2\int \frac{1}{x} dx \frac{1}{x} + c$

b. $\int_0^4 (\sqrt{x} + \frac{1}{\sqrt{x}}) dx$

$= \int_0^4 x^{1/2} + x^{-1/2}$

$= \frac{2}{3} x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c \Big|_0^4$

$= (\frac{2}{3}(4)^{\frac{3}{2}} + 2(4)^{\frac{1}{2}}) - (\frac{2}{3}(0)^{3/2})(\frac{28}{3}) - (0)$

$= \frac{28}{3}$

Nama : Agus Putut
NIM : 202710030

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c. $\int x^2 \sin x^3 \, dx$

$U = x^3$

$du = 3x^2 \, dx$

$\frac{1}{3} \, du = x^2 \, dx$

$\int x^2 . \sin x^3 \, dx = \frac{1}{3} (\sin (u) \, . du$

$= \frac{1}{3} (-\cos (u) + c$

$= \frac{1}{3} \cos x^3 + c$

d. $\int 3x (x^2 + 9)^5 \, dx$

misal : $u = x^2 + 5$

$du = 2x$

$\frac{1}{2} \, du = x$

$\int 3x (x^2 - 5) \, dx = 3 \int x (x^2 + 5)^5 \, dx$

$= 3 \, \frac{1}{2} \int (u)^5 \, dx$

$= \frac{3}{2} \cdot \frac{1}{6} (4)^6 + c$

$= \frac{(x^2 + 5)^6}{4} + c$

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NIM : 202710030

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c. $\int x^2 \sin x^3 \, dx$

$U = x^3$

$du = 3x^2 \, dx$

$\frac{1}{3} du = x^2 \, dx$

$\int x^2 . \sin x^3 \, dx = \frac{1}{3} \int \sin(u) . du$

$= \frac{1}{3} (-\cos(u) + c$

$= \frac{1}{3} \cos x^3 + c$

d. $\int 3x (x^2 + 5)^5 \, dx$

misal : $U = x^2 + 5$

$du = 2x$

$\frac{1}{2} du = x$

$\int 3x (x^2 - 5) \, dx = 3 \int x (x^2 + 5)^5 dx$

$= 3 \frac{1}{2} \int (u)^5 \, dx$

$= \frac{3}{2} . \frac{1}{6} (4)^6 + c$

$= \frac{(x^2 + 5)^6}{4} + c$

c. $\int x^2 \sin x^3 \, dx$

$u = x^3$

$du = 3x^2 \, dx$

$\frac{1}{3} du = x^2 \, dx$

$\int x^2 . \sin x^3 \, dx = \frac{1}{3} \int \sin(u) . du$

$= \frac{1}{3} (-\cos(u) + c$

$= \frac{1}{3} \cos x^3 + c$

d. $\int 3x (x^2 + 5)^5 \, dx$

misal : $u = x^2 + 5$

$du = 2x$

$\frac{1}{2} du = x$

$\int 3x (x^2 - 5) dx = 3 \int x (x^2 + 5)^5 dx$

$= 3 \frac{1}{2} \int (u)^5 \, dx$

$= \frac{3}{2} \cdot \frac{1}{6} (4)^6 + c$

$= \frac{(x^2 + 5)^6}{4} + c$

Nama : Agus Putut
NIM : 202710030

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(9) Suku pertama suatu deret geometri adalah 2 dan jumlah sampai tak terhingga adalah 4 carilah rasionya

jumlah = $\frac{a}{1-r}$

$4 = \frac{2}{1-r}$

$4(1-r) = 2$

$4 - 4r = 2$

$-4r = -2$

$r = \frac{-2}{-4}$

$= \frac{1}{2}$

(10) carilah n terkecil sehingga $S_n > 1.000$ pada deret geometri

$1 + 4 + 16 + 64 + \cdots$

$S_n = \frac{a(r^n - 1)}{r-1} = \frac{1(4^n - 1)}{4-1} = \frac{4^n - 1}{3}$

nilai n yang mengakibatkan $S_n > 1000$ adalah

$\frac{4^n - 1}{3} > 1.000 \Longleftrightarrow 4^n > 3.001$

jika dua ruas dilogaritmakan di peroleh.

$\log 4^n > \log . 3000$

$\Rightarrow n \log 4 > \frac{\log 3001}{\log 4}$

$n > 5,78$

Jadi nilai terkecil agar $S_n > 1.000$ adalah 6

Nama : Agus Putut
NIM : 202710030

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tentukan deret taylor dan deret mac lauren

$f(x) = e^x . \cos(x)$ sampai suku ke lima

$f'(x) = \frac{d}{dx}(e^x x \cos(x))$

$f'(x) = \frac{d}{dx}(e^x . x \cos(5x))$

gunakan aturan deferensial

$\frac{d}{dx}(f \times g) = \frac{d}{dx}(f) \times g + f \times \frac{d}{dx}(g)$

$f(x) = \frac{d}{dx}(e^x) \times \cos(5x) + e^x \times \frac{d}{dx} \cos(5x)$

$f'(x)\ e x \cos(5x) + ex \frac{d}{dx}(\cos(5x))$

$f'(x) = e x \cos(5x) - e5^x x \sin(5x)$

$f'(x) = e x \cos(5x) - e5^x \sin(5x)$